

## Contents

|   |                 |
|---|-----------------|
| <i>Preface</i>                                      | <i>page</i> vii |
| <i>Prerequisites</i>                                | 1               |
| 1 Foliations  | 4               |
| 1.1 Definition and first examples                   | 5               |
| 1.2 Alternative definitions of foliations           | 9               |
| 1.3 Constructions of foliations                     | 14              |
| 2 Holonomy and stability                            | 19              |
| 2.1 Holonomy  | 20              |
| 2.2 Riemannian foliations                           | 25              |
| 2.3 Local Reeb stability                            | 30              |
| 2.4 Orbifolds                                       | 34              |
| 2.5 Global Reeb stability in codimension 1          | 44              |
| 2.6 Thurston's stability theorem                    | 49              |
| 3 Two classical theorems                            | 56              |
| 3.1 Haefliger's theorem                             | 57              |
| 3.1.1 Review of Morse functions                     | 58              |
| 3.1.2 Morse functions into codimension 1 foliations | 60              |
| 3.1.3 Proof of Haefliger's theorem                  | 62              |
| 3.2 Novikov's theorem                               | 65              |
| 3.2.1 Vanishing cycles                              | 66              |
| 3.2.2 Existence of a compact leaf                   | 71              |
| 3.2.3 Existence of a Reeb component                 | 77              |
| 4 Molino's theory                                   | 81              |
| 4.1 Transverse parallelizability                    | 82              |
| 4.1.1 Homogeneous foliations                        | 82              |
| 4.1.2 Transversely parallelizable foliations        | 86              |

|       |   |     |
|-------|---|-----|
| vi    | <i>Contents</i>                           |     |
| 4.2   | Principal bundles                         | 92  |
| 4.2.1 | Connections on principal bundles          | 93  |
| 4.2.2 | Transverse principal bundles              | 98  |
| 4.3   | Lie foliations and Molino's theorem       | 101 |
| 4.3.1 | Lie foliations                            | 102 |
| 4.3.2 | The Darboux cover                         | 103 |
| 4.3.3 | Molino's structure theorem                | 108 |
| 5     | Lie groupoids                             | 110 |
| 5.1   | Definition and first examples             | 111 |
| 5.2   | The monodromy and holonomy groupoids      | 117 |
| 5.3   | Some general constructions                | 121 |
| 5.4   | Equivalence of Lie groupoids              | 127 |
| 5.5   | Étale groupoids                           | 134 |
| 5.6   | Proper groupoids and orbifolds            | 140 |
| 5.7   | Principal bundles over Lie groupoids      | 144 |
| 6     | Lie algebroids                            | 149 |
| 6.1   | The Lie algebroid of a Lie groupoid       | 150 |
| 6.2   | Definition and examples of Lie algebroids | 153 |
| 6.3   | Lie theory for Lie groupoids              | 157 |
| 6.4   | Integrability and developable foliations  | 160 |
|       | <i>References and further reading</i>     | 166 |
|       | <i>Index</i>                              | 170 |