INTRODUCTION TO FOLIATIONS AND LIE GROUPOIDS

This book gives a quick introduction to the theory of foliations, Lie groupoids, and Lie algebroids. An important feature is the emphasis on the interplay between these concepts; Lie groupoids form an indispensable tool for the study of the transverse structure of foliations as well as their non-commutative geometry, while the theory of foliations has immediate applications to the Lie theory of groupoids and their infinitesimal algebroids.

This book starts with a detailed presentation of the main classical theorems in the theory of foliations, then proceeds to Molino's theory, Lie groupoids, constructing the holonomy groupoid of a foliation, and finally Lie algebroids. Among other things, the authors discuss to what extent Lie's theory for Lie groups and Lie algebras holds in the more general context of groupoids and algebroids. Based on the authors' extensive teaching experience, this book contains numerous examples and exercises, making it ideal for graduate students and their instructors.

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I. Moerdijk , J. Mrcun
Frontmatter
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Preface

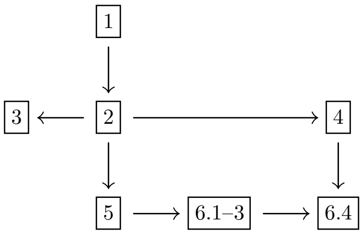
The purpose of this book is to give a quick introduction to the theory of foliations, as well as to Lie groupoids and their infinitesimal version – Lie algebroids. The book is written for students who are familiar with the basic concepts of differential geometry, and all the results presented in this book are proved in detail.

The topics in this book have been chosen so as to emphasize the relations between foliations, Lie groupoids and Lie algebroids. Lie groupoids form the main tool for the study of the 'transversal structure' (the space of leaves) of a foliation, by means of its holonomy groupoid. Foliations are also a special kind of Lie algebroids. At the same time, the elementary theory of foliations is a very useful tool in studying Lie groupoids and Lie algebroids.

In Chapter 1 we present the basic definitions, examples and constructions of foliations. Chapter 2 introduces the notion of *holonomy*, which plays a central role in this book. The Reeb stability theorems are discussed, as well as Riemannian foliations and their holonomy. This chapter also contains an introduction to the theory of orbifolds (or V-manifolds). Orbifolds provide a language to describe the richer structure of the space of leaves of certain foliations; e.g. the space of leaves of a Riemannian foliation is often an orbifold.

In Chapter 3 we present two classical milestones of the theory of foliations in codimension 1, namely the theorems of Haefliger and Novikov, with detailed proofs. Although the proofs make essential use of the notion of holonomy, this chapter is somewhat independent of the rest of the book (see the figure). However, it should be pointed out that there are proofs of Haefliger's theorem which use the transverse structure and Lie groupoids, see e.g. Jekel (1976) or Van Est (1984).

In Chapter 4, we discuss homogeneous and transversely parallelizable foliations, as well as Lie foliations, culminating in Molino's structure theorem



Interdependence of the chapters

for Riemannian foliations. This chapter also provides an essential link to the integrability theory of Lie groupoids and Lie algebroids.

In Chapter 5, we introduce the notion of Lie groupoid. The fine structure of the space of leaves of a foliation can be modelled by its holonomy and monodromy groupoids, and these provide some of the main examples of Lie groupoids. These Lie groupoids play an important role in the study of foliations from the point of view of non-commutative geometry as well; see Connes (1994). Orbifolds can also be viewed as Lie groupoids; in fact, they are shown to be essentially equivalent to a special class of Lie groupoids.

The infinitesimal part of a Lie groupoid gives rise to the structure of a Lie algebroid, similarly to the case of Lie groups and Lie algebras. In Chapter 6, we introduce these structures, and examine to what extent the correspondence between Lie groups and Lie algebras ('Lie theory') extends to Lie groupoids and Lie algebroids. Here we make essential use of elementary foliation theory, e.g. to construct the simply connected cover of a Lie groupoid, and to establish the correspondence of maps between Lie groupoids and maps between their Lie algebroids. Transversely parallelizable foliations from Chapter 4 provide natural examples of Lie algebroids which are not 'integrable', i.e. are not the infinitesimal parts of Lie groupoids.

This small book came into existence over a relatively long period of time. Chapters 1–3 are based on the notes of part of a course on foliations given at Utrecht University in 1995 and several subsequent years, and at the University of Ljubljana in 1997 and 1998. Chapter 4 was added later, in 1999. The last two chapters, on Lie groupoids and Lie algebroids, have been written more recently (in 2000) in this form, although much of this material had been presented by both of us in many earlier lectures and research papers.

Over the years, we have been influenced and helped by discussions with many friends and colleagues, and it would be impossible to thank them all here. However, we do wish to acknowledge our gratitude to A. Haefliger, who has been very encouraging, while it is obvious from the text that we owe a lot to his work. We are also much indebted to the late W.T. van Est who first got us interested in foliations, and to K.C.H. Mackenzie for many helpful discussions about Lie algebroids.

At a different level, we would like to thank the Dutch Science Foundation (NWO) and the Slovenian Ministry of Science (MŠZŠ grant J1-3148) for their support, as well as our home institutions for their support and hospitality, which made mutual visits possible and pleasant. Finally, we would like to thank the staff of Cambridge University Press for their help during the final stages of the preparation of this book.