

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels

Frontmatter

[More information](#)

---

This book is based on expository lectures by six internationally known experts presented at the 2002 MSRI introductory workshop on commutative algebra. They focus on the interaction of commutative algebra with other areas of mathematics, including algebraic geometry, group cohomology and representation theory, and combinatorics, with all necessary background provided. Short complementary papers describing work at the research frontier are also included. The unusual scope and format make the book invaluable reading for graduate students and researchers interested in commutative algebra and its various uses.

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels

Frontmatter

[More information](#)

---

Mathematical Sciences Research Institute  
Publications

51

---

Trends in Commutative Algebra

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels

Frontmatter

[More information](#)


---

## Mathematical Sciences Research Institute Publications

---

- 1 Freed/Uhlenbeck: *Instantons and Four-Manifolds*, second edition
- 2 Chern (ed.): *Seminar on Nonlinear Partial Differential Equations*
- 3 Lepowsky/Mandelstam/Singer (eds.): *Vertex Operators in Mathematics and Physics*
- 4 Kac (ed.): *Infinite Dimensional Groups with Applications*
- 5 Blackadar: *K-Theory for Operator Algebras*, second edition
- 6 Moore (ed.): *Group Representations, Ergodic Theory, Operator Algebras, and Mathematical Physics*
- 7 Chorin/Majda (eds.): *Wave Motion: Theory, Modelling, and Computation*
- 8 Gersten (ed.): *Essays in Group Theory*
- 9 Moore/Schochet: *Global Analysis on Foliated Spaces*
- 10–11 Drasin/Earle/Gehring/Kra/Marden (eds.): *Holomorphic Functions and Moduli*
- 12–13 Ni/Peletier/Serrin (eds.): *Nonlinear Diffusion Equations and Their Equilibrium States*
- 14 Goodman/de la Harpe/Jones: *Coxeter Graphs and Towers of Algebras*
- 15 Hochster/Huneke/Sally (eds.): *Commutative Algebra*
- 16 Ihara/Ribet/Serre (eds.): *Galois Groups over  $\mathbb{Q}$*
- 17 Concus/Finn/Hoffman (eds.): *Geometric Analysis and Computer Graphics*
- 18 Bryant/Chern/Gardner/Goldschmidt/Griffiths: *Exterior Differential Systems*
- 19 Alperin (ed.): *Arboreal Group Theory*
- 20 Dazard/Weinstein (eds.): *Symplectic Geometry, Groupoids, and Integrable Systems*
- 21 Moschovakis (ed.): *Logic from Computer Science*
- 22 Ratiu (ed.): *The Geometry of Hamiltonian Systems*
- 23 Baumslag/Miller (eds.): *Algorithms and Classification in Combinatorial Group Theory*
- 24 Montgomery/Small (eds.): *Noncommutative Rings*
- 25 Akbulut/King: *Topology of Real Algebraic Sets*
- 26 Judah/Just/Woodin (eds.): *Set Theory of the Continuum*
- 27 Carlsson/Cohen/Hsiang/Jones (eds.): *Algebraic Topology and Its Applications*
- 28 Clemens/Kollár (eds.): *Current Topics in Complex Algebraic Geometry*
- 29 Nowakowski (ed.): *Games of No Chance*
- 30 Grove/Petersen (eds.): *Comparison Geometry*
- 31 Levy (ed.): *Flavors of Geometry*
- 32 Cecil/Chern (eds.): *Tight and Taut Submanifolds*
- 33 Axler/McCarthy/Sarason (eds.): *Holomorphic Spaces*
- 34 Ball/Milman (eds.): *Convex Geometric Analysis*
- 35 Levy (ed.): *The Eightfold Way*
- 36 Gavosto/Krantz/McCallum (eds.): *Contemporary Issues in Mathematics Education*
- 37 Schneider/Siu (eds.): *Several Complex Variables*
- 38 Billera/Björner/Green/Simion/Stanley (eds.): *New Perspectives in Geometric Combinatorics*
- 39 Haskell/Pillay/Steinhorn (eds.): *Model Theory, Algebra, and Geometry*
- 40 Bleher/Its (eds.): *Random Matrix Models and Their Applications*
- 41 Schneps (ed.): *Galois Groups and Fundamental Groups*
- 42 Nowakowski (ed.): *More Games of No Chance*
- 43 Montgomery/Schneider (eds.): *New Directions in Hopf Algebras*
- 44 Buhler/Stevenhagen (eds.): *Algorithmic Number Theory*
- 45 Jensen/Ledet/Yui: *Generic Polynomials: Constructive Aspects of the Inverse Galois Problem*
- 46 Rockmore/Healy (eds.): *Modern Signal Processing*
- 47 Uhlmann (ed.): *Inside Out: Inverse Problems and Applications*
- 48 Gross/Kotiuga: *Electromagnetic Theory and Computation: A Topological Approach*
- 49 Darmon/Zhang (eds.): *Heegner Points and Rankin L-Series*
- 50 Bao/Bryant/Chern/Shen (eds.): *A Sampler of Riemann–Finsler Geometry*
- 51 Avramov/Green/Huneke/Smith/Sturmfels (eds.): *Trends in Commutative Algebra*

Volumes 1–4 and 6–27 are published by Springer-Verlag

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels

Frontmatter

[More information](#)

# Trends in Commutative Algebra

*Edited by*

**Luchezar L. Avramov**

*University of Nebraska*

**Mark Green**

*University of California, Los Angeles*

**Craig Huneke**

*University of Kansas*

**Karen E. Smith**

*University of Michigan*

**Bernd Sturmfels**

*University of California, Berkeley*



**CAMBRIDGE**  
UNIVERSITY PRESS

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels

Frontmatter

[More information](#)

Luchezar L. Avramov  
University of Nebraska

Mark Green  
Univ. of California, Los Angeles

Craig Huneke  
University of Kansas

Karen E. Smith  
University of Michigan

Bernd Sturmfels  
Univ. of California, Berkeley

*Series Editor*

Silvio Levy  
Mathematical Sciences  
Research Institute  
17 Gauss Way  
Berkeley, CA 94720  
United States

*MSRI Editorial Committee*

Hugo Rossi (chair)  
Alexandre Chorin  
Silvio Levy  
Jill Mesirov  
Robert Osserman  
Peter Sarnak

The Mathematical Sciences Research Institute wishes to acknowledge support by the National Science Foundation. This material is based upon work supported by NSF Grant 9810361.

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS  
The Edinburgh Building, Cambridge CB2 2RU, UK  
40 West 20th Street, New York, NY 10011-4211, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
Ruiz de Alarcón 13, 28014 Madrid, Spain  
Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

© Mathematical Sciences Research Institute 2004

First published 2004

Printed in the United States of America

*A catalogue record for this book is available from the British Library.*

*Library of Congress Cataloging in Publication data*

Trends in commutative algebra / edited by Luchezar L. Avramov ... [et al.]  
p. cm.

(Mathematical Sciences Research Institute publications ; 51)

Includes bibliographical references and index.

ISBN 0-521-83195-4 (hb)

1. Commutative algebra. I. Avramov, L. L. (Luchezar L.), 1948–  
II. Title. III. Series.

QA251.3.T74 2005

512'.44-dc22 2004022712

ISBN 0 521 83195 4 hardback

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels

Frontmatter

[More information](#)Trends in Commutative Algebra  
MSRI Publications  
Volume 51, 2004

## Contents

Preface	ix
Commutative Algebra in the Cohomology of Groups	1
DAVE BENSON	
Modules and Cohomology over Group Algebras	51
SRIKANTH IYENGAR	
An Informal Introduction to Multiplier Ideals	87
MANUEL BLICKLE AND ROBERT LAZARSFELD	
Lectures on the Geometry of Syzygies	115
DAVID EISENBUD, with a chapter by JESSICA SIDMAN	
Commutative Algebra of $n$ Points in the Plane	153
MARK HAIMAN, with an appendix by EZRA MILLER	
Tight Closure Theory and Characteristic $p$ Methods	181
MELVIN HOCHSTER, with an appendix by GRAHAM J. LEUSCHKE	
Monomial Ideals, Binomial Ideals, Polynomial Ideals	211
BERNARD TEISSIER	
Some Facts About Canonical Subalgebra Bases	247
ANA BRAVO	

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels

Frontmatter

[More information](#)Trends in Commutative Algebra  
MSRI Publications  
Volume 51, 2004

## Preface

Over the last fifteen years, commutative algebra has experienced a striking evolution. During this period the outlook of the subject has been altered, new connections to other areas have been established, and powerful techniques have been developed. To foster further development a year-long program on commutative algebra was held at MSRI during the 2002–03 academic year, starting with an introductory workshop on September 9–13, 2002. This workshop concentrated on the interplay and growing connections between commutative algebra and other areas, such as algebraic geometry, the cohomology of groups, and combinatorics.

Six main speakers each gave a series of three talks during the week: David Benson, David Eisenbud, Mark Haiman, Melvin Hochster, Rob Lazarsfeld, and Bernard Teissier. The workshop was very well attended, with more than 120 participants. Every series of main talks was supplemented by a discussion/talk session presented by a young researcher: Manuel Blickle, Ana Bravo, Srikanth Iyengar, Graham Leuschke, Ezra Miller, and Jessica Sidman. Each of these speakers has contributed a paper, or in some cases a combined paper, in this volume.

David Benson spoke on the cohomology of groups, presenting some of the many questions which are unanswered and which have a close relationship to modern commutative algebra. He gave us many convincing reasons for working in the “graded” commutative case, where signs are introduced when commuting elements of odd degree. Srikanth Iyengar gives background information for Benson’s notes.

David Eisenbud spoke on a classical subject in commutative algebra: free resolutions. In his paper with a chapter by Jessica Sidman, he visits this classic territory with a different perspective, by drawing close ties between graded free resolutions and the geometry of projective varieties. He leads us through recent developments, including Mark Green’s proof of the linear syzygy conjecture.

Mark Haiman lectured on the commutative algebra of  $n$  points in the plane. This leads quite rapidly to the geometry of the Hilbert scheme, and to substantial combinatorial questions (and answers) which can be phrased in terms of common questions in commutative algebra such as asking about the Cohen–Macaulay

Cambridge University Press

0521831954 - Trends in Commutative Algebra

Edited by Luchezar L. Avramov, Mark Green, Craig Huneke, Karen E. Smith and Bernd Sturmfels  
Frontmatter[More information](#)

property for certain Rees algebras. Ezra Miller writes an appendix about the Hilbert scheme of  $n$  points in the plane.

Mel Hochster gave three lectures on tight closure, telling eleven reasons to study tight closure. Hochster presents tight closure as a test for ideal membership which is necessary, but not sufficient, except for certain rings such as regular rings. Graham Leuschke's appendix gives examples of computation of tight closure.

The theory of multiplier ideals has been expanding rapidly in the last few years and bears a close relationship to commutative algebra, particularly tight closure. Rob Lazarsfeld and Manuel Blickle present a gentle introduction to this theory, with emphasis on the important theorems and concepts, applications, and examples.

Resolution of singularities has long played a crucial role in algebraic geometry and commutative algebra. Bernard Teissier talked about new ideas for understanding resolution coming from the simplest of all polynomials: monomials and binomials. Toric geometry of course enters into this story in a crucial way. Ana Bravo provides a summary of results on SAGBI bases which enter into this story.

The editors of this volume, who formed the organizing committee for the year program, would like to thank the many people who made the year possible, and thank the speakers for their wonderful contributions. A special thanks to David Eisenbud, the director of MSRI, without whom none of this would have been possible. We thank Michael Singer, the acting director of MSRI during the academic year when the program took place, for his generous help, and for the loan of Eisenbud to participate in our program. The great staff at MSRI were unfailingly helpful, friendly and professional. We thank the MSRI editor, Silvio Levy, for all his work on this volume. Finally, we thank the National Science Foundation for its support of institutes of mathematics in general, and of MSRI in particular.

We hope the papers in this volume will be a springboard for further learning and research for both experts and beginners.

Luchezar Avramov  
Mark Green  
Craig Huneke  
Karen Smith  
Bernd Sturmfels

**Note:** The lectures this volume is based on were videotaped. They are available on streaming video and for downloading at [www.msri.org/publications/video](http://www.msri.org/publications/video) or [www.cambridge.org/0521831954](http://www.cambridge.org/0521831954).