Cambridge University Press
0521831814 - A Sampler of Riemann-Finsler Geometry
Edited by David Bao, Robert L. Bryant, Shiing-Shen Chern and Zhongmin Shen
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Finsler geometry generalizes Riemannian geometry in the same sense that Banach spaces generalize Hilbert spaces. This book presents an expository account of seven important topics in Riemann–Finsler geometry, which have recently undergone significant development but have not had a detailed pedagogical treatment elsewhere. Each article will open the door to an active area of research and is suitable for a special topics course in graduate-level differential geometry.

Álvarez and Thompson discuss the theory of volumes for normed spaces and Finsler spaces and show how it unifies a wide range of geometric inequalities. Bellettini studies the evolution of crystals, where the driving agent is the mean curvature of the facets. Aikou reviews the essential role played by Finsler metrics in complex differential geometry. Chandler and Wong explain why parametrized jet bundles admit only Finsler metrics and develop machinery which they use to prove the Kobayashi conjecture (1960) and a special case of the Green–Griffiths (1979) conjecture. Bao and Robles focus on the flag and Ricci curvatures of Finsler manifolds, with an emphasis on Einstein metrics of Randers type. Rademacher gives a detailed and new account of his Sphere Theorem for nonreversible Finsler metrics. Shen's article explains why Finsler manifolds are colorful objects and examines the interplay among the flag, S-, and Landsberg curvatures in Finsler geometry.

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A Sampler of Riemann–Finsler Geometry

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Edited by

David Bao
University of Houston

Robert L. Bryant
Duke University

Shiing-Shen Chern
UC Berkeley and Nankai Institute of Mathematics

Zhongmin Shen
Indiana University – Purdue University Indianapolis

CAMBRIDGE
UNIVERSITY PRESS

David Bao
Department of Mathematics
University of Houston
Houston, TX 77204-3008
United States
bao@math.uh.edu

Robert L. Bryant
Duke University

Shiing-Shen Chern
Nankai Institute of Mathematics

Zhongmin Shen
IUPUI

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Preface

This volume contains seven expository articles and concerns three facets of Riemann–Finsler geometry that have undergone important recent developments:

1. The concept of volumes on normed spaces and Finsler manifolds, and crystalline motion by mean curvature in phase transitions.
2. The essential role played by Finsler metrics in complex manifold theory, together with the resolution of the Kobayashi conjecture and a special case of the Green–Griffiths conjecture.
3. The significance of the flag, Ricci, and S-curvatures of Finsler metrics, as well as the Sphere Theorem for nonreversible Finsler structures.

Conspicuously absent from the above are two highly geometrical areas: Bryant's use of exterior differential systems to understand Finsler metrics of constant flag curvature, and Foulon's dynamical systems approach to Finsler geometry. They are not included here because reasonable expositions already exist in a special Chern issue of the *Houston Journal of Mathematics* **28** (2002), 221–262 (Bryant) and 263–292 (Foulon).

Our goal is to render the aforementioned developments accessible to the differential geometry community at large. It is not our intention to present an encyclopedic picture of the field. What we do covet are concrete examples, instructive graphics, meaningful computations, and care in organizing technical arguments. The resulting articles appear to have met these criteria at an above-average level.

All the articles have been refereed. In fact, a total of 26 referee reports were obtained, some addressing the mathematics, others critiquing expository matters. After a few rounds of revision, each article was line-edited by at least one mathematician who is *not* familiar with the topic in question, in the hope that this would uncover most typographical mistakes. It is certain that, in spite of these attempts at quality control, layers of blemishes are still awaiting their turn to surface. We urge readers to bring those mistakes to the attention of one of us (Bao).

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David Bao, Houston, USA
Robert L. Bryant, Durham, USA
Shiing-Shen Chern, Nankai, China
Zhongmin Shen, Indianapolis, USA

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Synopses

1. Part One

- **Álvarez–Thompson**: This paper is a concise introduction to the theory of volumes on normed and Finsler spaces. The definitions of Holmes–Thompson, Busemann, and Benson–Gromov are studied and their convexity (ellipticity) properties are discussed in detail. The authors show how the theory of volumes provides a unified context for a diverse range of geometric inequalities. The article is intended for students and researchers in differential, integral, and convex geometry.
- **Bellettini**: Crystalline motion driven by the mean curvature is an evolution process arising in material science and phase transitions. It is an anisotropic flow in an ambient space endowed with a piecewise linear norm. For three-dimensional crystals, the crystalline mean curvature of a facet is defined and identified with the initial velocity in the evolution process. Facets with constant crystalline mean curvature are important because they are expected not to break or curve under the evolution. The problem of characterizing such facets is discussed.

2. Part Two

- **Aikou**: This article highlights the essential role played by Finsler metrics in complex differential geometry. It describes a few situations for which techniques based solely on Hermitian metrics are hopelessly inadequate. These include Kobayashi's characterization of negative holomorphic vector bundles over compact complex manifolds, in terms of the existence of negatively curved pseudoconvex Finsler metrics.
- **Chandler–Wong**: The authors present the proof of the Kobayashi conjecture (1960) on the hyperbolicity of generic algebraic surfaces of degree $d \geqslant 5$ in $\mathbb{P}^3$. They also address the Green–Griffiths conjecture (1979) that every holomorphic map $f : \mathbb{C} \to X$ to a surface X of general type is algebraically degenerate. Their paper establishes the latter for the *special* case where X is

minimal, $\mathrm{Pic}(X) \cong \mathbb{Z}$, and $p_g(X) > 0$. The main tool used is their generalization of the classical Schwarz lemma for complex curves, to varieties of every dimension. In this crucial step, algebraic geometric arguments are used to construct a Finsler metric of logarithmic type, thereby reducing the problem to one in which a certain estimate (the lemma of logarithmic derivatives) is applicable.

3. Part Three

- **Bao–Robles**: Many recent developments have advanced our understanding of the flag and Ricci curvatures of Finsler metrics. This paper is a uniform presentation of these results and their underlying techniques. Included is a geometric definition of Einstein–Finsler metrics. Einstein metrics of Randers type are studied via their representation as solutions to Zermelo navigation on Riemannian manifolds. This viewpoint leads to the classification of *all* constant flag curvature Randers metrics. It also yields a Schur lemma, and settles a question of rigidity in three dimensions, for Einstein–Randers metrics. The theory is illustrated with a diverse array of explicit examples.
- **Rademacher**: The author shows in detail how the classical Sphere Theorem in Riemannian geometry is extended to the case of nonreversible Finsler metrics. The proof hinges on a fruitful definition of the notion of reversibility, and how that can be used to effect some crucial estimates, such as the injectivity radius, the length of nonminimal geodesics between two fixed points, and the length of nonconstant geodesic loops. The proof also capitalizes on an idea of Klingenberg: that Morse theory of the energy functional allows us to circumvent Toponogov's comparison theorem for geodesic triangles. This idea renders irrelevant the "handicap" that, in Finsler geometry proper, there is no Toponogov's theorem.
- **Shen**: This paper is about the interaction between the generalized Riemann curvature and other non-Riemannian quantities in Finsler geometry. The latter include the S-curvature, whose vanishing is equivalent to having constant "distortion" along each geodesic. The S-curvature quantifies some aspect of the change in the Minkowski model ("infinitesimal color pattern") as one moves from one tangent space to another, along geodesics on a Finslerian landscape. The information it provides is complementary to that supplied by a certain contracted version of the Berwald curvature. These theoretical constructs are exemplified by Finsler metrics with a broad array of special curvature properties. The author also proves a number of local and global theorems using certain curvature equations along geodesics.