

## Index

- affine space 2
- affine variety 2
- annihilator 7,12,82
- antipodal point 21
- Artinian
  - ring 25
  - module 25, 103
  - reduction 100
- associated prime of ideal 8
  - of module 11
- Auslander-Buchsbaum theorem 148
  
- Betti numbers (of resolution) 38
  - and Tor 114
- Betti numbers (topological) 67
- Bezier cubic 2
- Bezout's theorem 20,23,31–32,37
- bilinear map 89
- boundary, of simplex 66
- Buchberger algorithm 53
  
- Castelnuovo-Mumford regularity
  - see regularity
- canonical divisor 139
- category 80
- catenary 146
- Cauchy-Riemann equations 175–176
- Cauchy's theorem 178
- Cauchy integral formula 179
- Cech complex 130
- chain, ascending 4
  - descending 25
- chain complex 27
- chain homotopy 112
- CoCoA 174
- codimension
  - of ideal 146
  - of projective variety 29
- coface 78
- Cohen-Macaulay 149
  - and codimension 150
  - and Ext 152
  - and Stanley-Reisner ring 159
- arithmetically 149
- module 149
- ring 149
- cohomology
  - Cech 130–132, 162
  - local 161–162
  - sheaf 129–132
  - simplicial 86
- commutative diagram 35, 107
- complete intersection 32, 46
  - local 82–84
  - resolution of 46–48
- complex, chain 27
  - cellular 72
  - simplicial 66
- complex number 175
- contravariant 80, 116
- convex combination 2,73
- convex set 2
- coordinate ring 13, 155
- covariant 80, 112, 116
  
- decomposition
  - primary 6
  - irreducible 6
- degree
  - and Hilbert polynomial 29–31
  - and hyperplane section 42–45
  - of polynomial 22
  - of projective variety 29–31
  - shifting 22
- Dehn-Sommerville relations 76,158
- depth 147–148

- dimension
  - and Hilbert polynomial 29–31
  - and hyperplane section 42–45
  - of graded piece  $M_i$  22
  - of module 147
  - of projective variety 29–31
  - of ring (Krull) 146
  - relation among definitions 147
- direct limit 168, 127
- division algorithm 52
- divisor 133–143
  - and locally free sheaf 134
  - and global sections 135
  - effective 141
  - map associated to 135
  - nonspecial 142
  - very ample 135
- double complex 124
  
- Eisenbud-Goto conjecture 158
- elimination ideal 60–61
- embedding 133
- Euclidean algorithm 168, 50
- Euler characteristic 27, 140
  - relation 73
- exact sequence 27
  - and nonzerodivisor 42
  - of complexes 108
  - of sheaf cohomology 131
- exponent vector 51
- Ext 116–123
  - and associated primes 118, 152
  - and localization 152
- extension of
  - modules 123
  - scalars 89
  
- f-vector 73, 158–161
- field 164
- free resolution 36–42
  - and generic points 97–98
  - and Hilbert series 41
  - finite 37
  - graded 37
  - infinite 40
  - minimal 40
- functor 80
  - additive 112
  - contravariant 80, 116
  - covariant 80, 112
  - exact 81
  - left derived 111–112
  - left exact 85, 116
  - right derived 116
  - right exact 90, 112
- generic condition 33
  - hyperplane 29, 31
  - points 97–98
- genus
  - of curve 134, 139, 173
  - of real surface 111
  - of simplicial complex 111
- global section 128–132
- Gotzmann's theorem 99
- graded
  - lex order 51
  - map 26
  - module 22
  - ring 21
- Green's conjecture 143
- Green's theorem 176–178
- Gröbner basis 53
  - and modules 58–60
  - and elimination 60–63
  - and syzygies 59–60
- group 163–164
  - abelian 163
  - homomorphism 164
  - normal subgroup 164
  
- h-vector 73–76, 159–60
- Hilbert basis theorem 5
- Hilbert-Burch theorem 150–152
- Hilbert function 22
  - and points 92–98
  - growth of 99–106
  - of polynomial ring 22
- Hilbert nullstellensatz 14
- Hilbert polynomial 28–31
  - and monomial ideal 55–57
  - and resolution 41
  - and Riemann-Roch 140–141
  - of points 29
- Hilbert series 23
  - and resolution 41
  - of Artinian module 25
  - of polynomial ring 25
- Hilbert syzygy theorem 37
  - proof of 114
- holomorphic 176
- Hom 80, 84–88
  - graded 88
- homogeneous coordinates 19

- homogeneous ideal 21
- homogeneous polynomial 20
- homology 27
  - long exact sequence 109
  - reduced 67
  - simplicial 67
- homomorphism
  - group 164
  - module 166
  - ring 166
  - sheaf 128
- Horseshoe lemma 115
- hyperelliptic 143
- hypersurface 2
- ideal 165
  - ascending chain of 4
  - generators of 2
  - initial 10
  - intersection 4
  - irreducible 167
  - maximal 167
  - membership 3
  - monomial 10
  - of a set 3
  - primary 167
  - prime 167
  - principal 167
  - product of 4
  - quotient 10
  - radical 7, 167
  - sum of 4
- independent conditions 93
- integral domain 165
- Jacobson radical 36
- Jacobian
  - ideal 82
  - matrix 171
- Klein bottle 71
- Koszul complex 47,60,114
- Krull dimension 146
- Laurent series 181, 134
- Leray cover 131
- lexicographic order 51
  - and projection 61–62
- line arrangement 82–84
- linear equivalence 135
- linear system
  - complete 143
  - incomplete 155–156
- local duality 155,161
  - and Ext 155
- localization 81–84
- Macaulay 2
  - getting started 168–171
  - commands 193
- Macaulay's theorem 99
- manifold 134,111
- mapping cone 47
- Mayer-Vietoris sequence 109–111
- meromorphic function 181, 134–139
- module, 165
  - annihilator of 82
  - Cohen-Macaulay 149
  - finitely generated 165
  - free 165, 35
  - graded 21,22
  - homomorphism 166
  - Noetherian 4
  - projective 35
- monomial
  - ideal 26
  - initial 51
  - order 51
- Motzkin's conjecture 158
- multiplicity 32–33, 20
- Nakayama's lemma 36
- Noetherian module 5
  - ring 4
- node 173
- nonzerodivisor 42
- nullstellensatz 14
- one-form 139
- orientation
  - of surface 111,125
  - of simplicial complex 66
- open set 13
  - distinguished 13
- Poincaré duality 111, 76
- Poincaré-Hopf theorem 73
- polytope 73
  - cyclic 158
- presheaf 126–127
- prime,
  - avoidance 44
  - embedded 8
  - minimal 8
- principal ideal domain 167–168,146

- projection 60–61
- projective
  - dimension 145
  - module 35
  - plane 19
  - space 18
  - variety 21
- projectively normal 143
- quasicompact 13
- rational quartic 153
  - and Riemann-Roch 153
  - cohomology of 155–156
- regular function 13, 15, 129
- regular map 14
- regular sequence 45, 147
- regularity 97, 156–158
- Reisner's theorem 159
- residue 182, 139–140
- resolution
  - free see free resolution
  - injective 116
  - projective 112
- Riemann-Hurwitz formula 174
- Riemann-Roch theorem 139–142
  - problem 135
- Riemann surface 134
- ring, 164–168
  - Cohen-Macaulay 149
  - commutative 164
  - graded 21
  - homomorphism 166
  - local 167, 174
  - Noetherian 4
  - regular local 174
  - Stanley-Reisner 72
- saturation 104
- Schlegel diagram 73
- Serre duality 141
- sheaf 126–129
  - associated 128
  - coherent 129
  - cohomology 129–132
    - and Ext 153–156
  - locally free 129
  - of modules 128
- simplex
  - abstract 65
  - boundary of 66
  - geometric 65
- oriented 65
- standard unit 65
- simplicial complex 66
  - oriented 66
- simplicial homology 66–67
- simplicial sphere 158–159
- simplicial polytope 73
- singular point 171
- Singular 174
- singularity, isolated 181
- smooth 171–174, 134
- snake lemma 107–108
- spectral sequence 125
- stalk 127
- Stanley-Reisner ring 72–79,
  - and Hilbert polynomial 76
  - and Hilbert series 74
  - and primary decomposition 77–79
- subvariety 3
- support
  - of module 82
  - of monomial 76
- surface,
  - genus of 110–111
  - orientable 111, 125
  - Riemann 134
- Taylor series 181, 128
- tensor product 88–91, 69
- Tor 112–116, 149
- torus 71
- topology 12
  - subspace 13
  - Zariski 13
- twisted cubic 12, 32, 39, 56, 60, 135–136, 150
- upper bound conjecture 158–161
- variety 2
  - irreducible 3
  - projective 21
- Veronese surface 137
- very ample 135
- Yoneda pairing 123
- Zariski closed set 13
  - closure 15
  - open set 13
  - topology 13
  - topology on  $P^n$  21
- zero-divisor 164–165

- Macaulay 2 Commands  
 apply 171,172  
 annihilator 120,121  
 basis 93,137  
 betti 38--39  
 chainComplex 69  
 codim 30,150,172  
 coker 23, 31  
 differentials (dd) 38--39  
 degree 31,172  
 degrees 170  
 equality (==) 9  
 Ext 117,119,155  
 factor 25  
 grobner basis (gb) 54  
 genericMatrix 57  
 gens 169, 31  
 graded Lex order 54  
 hilbertFunction 23  
 hilbertPolynomial 30  
 Hom 87  
 homology (HH<sub>i</sub>) 70  
 ideal 8  
 ideal quotient (I:J) 16  
 image 161  
 intersect 8  
 jacobian 83,172  
 kernel 169, 154,172  
 Lex order 62  
 load 48--49,173  
 local cohomology (HH<sup>i</sup>) 161  
 map 93,136,137,153  
 matrix 169, 23  
 mingens 172  
 minors 57,138,151,172  
 MonomialOrder 54  
 pdim 150  
 poincare 24  
 pfaffians 143  
 primaryDecomposition 9,  
 120  
 prune 87,88  
 radical 9  
 random 31,172  
 rank 70  
 res 38  
 ring 169  
 scan 171  
 source 170  
 submatrix 94  
 subquotient 87,88  
 target 170  
 tensor (\*\*) 69,91  
 time 57  
 Tor 113,114  
 transpose 94  
 vars 94  
 while 171