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Martin Charles Golumbic and Ann N. Trenk
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TOLERANCE GRAPHS

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Dedicated to
Lynn Pollak Golumbic and Rick Cleary

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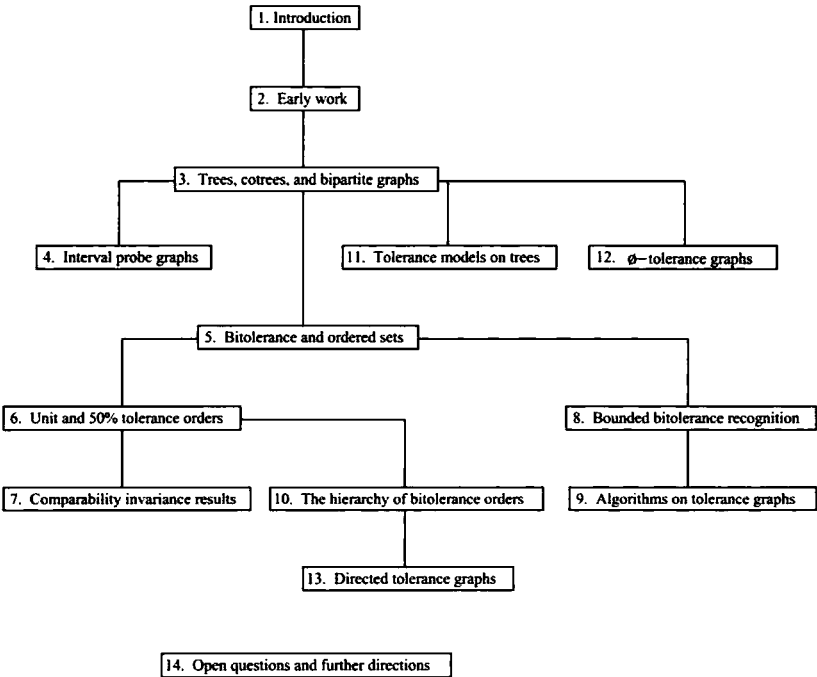
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Preface

At the 13th Southeastern Conference on Combinatorics, Graph Theory and Computing (Boca Raton, 1982), a mathematical model of tolerance, called *tolerance graphs*, was introduced by Golumbic and Monma in order to generalize some of the well known applications associated with interval graphs. Their motivation was the need to solve scheduling problems in which resources such as rooms, vehicles, support personnel, etc. may be required on an exclusive basis, but where a measure of flexibility or tolerance would be allowed for sharing or relinquishing the resource when total exclusivity prevented a solution. An example of such an application opens our Chapter 1.

During the ensuing years, properties of tolerance graphs have been studied, and quite a number of variations have appeared in the literature, including bitolerance graphs, bounded tolerance orders, NeST graphs, ϕ -tolerance graphs, tolerance digraphs and others. This continues to be an interesting and active area of investigation. At the 30th Southeastern Conference on Combinatorics, Graph Theory and Computing (Boca Raton, 1999), Ann delivered an invited survey talk on the subject, and together we organized a special session on tolerance graphs and related topics. The following year, Marty gave a largely complementary survey talk at the Fields Institute Workshop on Structured Families of Graphs (Toronto, 2000). In July 2001, DIMACS sponsored a workshop on Intersection Graphs and Tolerance Graphs.

It seems to us that the time is ripe to collect and survey the major results on tolerance graphs, presenting them in one volume. Many mathematical scientists around the world have carried out the research which has enabled us to do this, and we salute them. Yet there are still various basic unanswered questions concerning tolerance graphs. Tolerance graphs have not yet been characterized, nor are there recognition algorithms. Other open questions appear in Chapter 14. We hope that this book helps to inspire others to pursue these topics further.

What started as a survey paper has grown into a three year project and a 300-page manuscript. Even so, we have had neither time nor space to include all the topics we would have liked. In particular, we have not covered interval digraphs or the literature on tolerance competition graphs, or very recent results which have not had the opportunity to appear in a journal.

This book is intended primarily for researchers and graduate students, although some of it should be accessible to advanced undergraduates. We have included exercises to facilitate the use of this book in a seminar course. Algorithms and applications are presented in addition to the theory of tolerance graphs. In general, we have tried to include proofs whenever possible, omitting them only when they already appear in other books or sometimes when they are quite long. In several chapters we include hierarchies of structured families of graphs. Naturally, we have tried to catch all errors. We hope our readers will be tolerant of those that inevitably remain, and will report these errors to us.

We would like to thank our families for their support in our project, especially our spouses Lynn and Rick to whom we dedicate this book. Ann's parents were very helpful in finding us places to work in New York City several times, and Marty's daughters demonstrated unbounded tolerance for his traveling too much. We would like to acknowledge several colleagues who have worked with us on tolerance problems over the years and during the writing of this monograph: Ken Bogart, Garth Isaak, Robert Jamison, Haim Kaplan, Marina Lipshteyn, Clyde Monma, Kathryn Nyman, Uri Peled, Ron Shamir, Randy Shull, Assaf Siani, and Tom Trotter. We are grateful to Mike Fisher for a thorough reading of a draft of our manuscript and to Peter Fishburn, Ross McConnell and Jerry Spinrad for useful comments, pointers, references and encouragement. Phil Hirschhorn, Greta Pangborn, and Michael Wagner provided invaluable computer assistance, and Wellesley College students Charlotte Henderson and Jue Wang were a tremendous help in the editing stage. We also thank the institutions which have provided some of the support for our joint work: The American Association of University Women, Bar-Ilan, Cornell, Haifa, Rutgers, and Wellesley.