

Contents

<i>Preface</i>	xi
1 Newtonian mechanics of a single particle	1
1 The algebra and calculus of vectors	3
1.1 Vectors and vector quantities	3
1.2 Linear operations: $\mathbf{a} + \mathbf{b}$ and $\lambda \mathbf{a}$	5
1.3 The scalar product $\mathbf{a} \cdot \mathbf{b}$	10
1.4 The vector product $\mathbf{a} \times \mathbf{b}$	13
1.5 Triple products	15
1.6 Vector functions of a scalar variable	16
1.7 Tangent and normal vectors to a curve	18
Problems	22
2 Velocity, acceleration and scalar angular velocity	25
2.1 Straight line motion of a particle	25
2.2 General motion of a particle	28
2.3 Particle motion in polar co-ordinates	32
2.4 Rigid body rotating about a fixed axis	36
2.5 Rigid body in planar motion	38
2.6 Reference frames in relative motion	40
Problems	43
3 Newton's laws of motion and the law of gravitation	50
3.1 Newton's laws of motion	50
3.2 Inertial frames and the law of inertia	52
3.3 The law of mutual interaction; mass and force	54
3.4 The law of multiple interactions	57
3.5 Centre of mass	58

vi	Contents
3.6	The law of gravitation 59
3.7	Gravitation by a distribution of mass 60
3.8	The principle of equivalence and g 67
	Problems 71
4	Problems in particle dynamics 73
4.1	Rectilinear motion in a force field 74
4.2	Constrained rectilinear motion 78
4.3	Motion through a resisting medium 82
4.4	Projectiles 88
4.5	Circular motion 92
	Problems 98
5	Linear oscillations 105
5.1	Body on a spring 105
5.2	Classical simple harmonic motion 107
5.3	Damped simple harmonic motion 109
5.4	Driven (forced) motion 112
5.5	A simple seismograph 120
5.6	Coupled oscillations and normal modes 121
	Problems 126
6	Energy conservation 131
6.1	The energy principle 131
6.2	Energy conservation in rectilinear motion 133
6.3	General features of rectilinear motion 136
6.4	Energy conservation in a conservative field 140
6.5	Energy conservation in constrained motion 145
	Problems 151
7	Orbits in a central field 155
7.1	The one-body problem – Newton’s equations 157
7.2	General nature of orbital motion 159
7.3	The path equation 164
7.4	Nearly circular orbits 167
7.5	The attractive inverse square field 170
7.6	Space travel – Hohmann transfer orbits 177
7.7	The repulsive inverse square field 179
7.8	Rutherford scattering 179
Appendix A	The geometry of conics 184
Appendix B	The Hohmann orbit is optimal 186
	Problems 188

Contents	vii
8 Non-linear oscillations and phase space	194
8.1 Periodic non-linear oscillations	194
8.2 The phase plane $((x_1, x_2)$ -plane)	199
8.3 The phase plane in dynamics $((x, v)$ -plane)	202
8.4 Poincaré-Bendixson theorem: limit cycles	205
8.5 Driven non-linear oscillations	211
Problems	214
2 Multi-particle systems	219
9 The energy principle	221
9.1 Configurations and degrees of freedom	221
9.2 The energy principle for a system	223
9.3 Energy conservation for a system	225
9.4 Kinetic energy of a rigid body	233
Problems	241
10 The linear momentum principle	245
10.1 Linear momentum	245
10.2 The linear momentum principle	246
10.3 Motion of the centre of mass	247
10.4 Conservation of linear momentum	250
10.5 Rocket motion	251
10.6 Collision theory	255
10.7 Collision processes in the zero-momentum frame	259
10.8 The two-body problem	264
10.9 Two-body scattering	269
10.10 Integrable mechanical systems	273
Appendix A Modelling bodies by particles	277
Problems	279
11 The angular momentum principle	286
11.1 The moment of a force	286
11.2 Angular momentum	289
11.3 Angular momentum of a rigid body	292
11.4 The angular momentum principle	294
11.5 Conservation of angular momentum	298
11.6 Planar rigid body motion	306
11.7 Rigid body statics in three dimensions	313
Problems	317

viii	Contents
3	Analytical mechanics 321
12	Lagrange's equations and conservation principles 323
12.1	Constraints and constraint forces 323
12.2	Generalised coordinates 325
12.3	Configuration space (q -space) 330
12.4	D'Alembert's principle 333
12.5	Lagrange's equations 335
12.6	Systems with moving constraints 344
12.7	The Lagrangian 348
12.8	The energy function h 351
12.9	Generalised momenta 354
12.10	Symmetry and conservation principles 356
	Problems 361
13	The calculus of variations and Hamilton's principle 366
13.1	Some typical minimisation problems 367
13.2	The Euler–Lagrange equation 369
13.3	Variational principles 380
13.4	Hamilton's principle 383
	Problems 388
14	Hamilton's equations and phase space 393
14.1	Systems of first order ODEs 393
14.2	Legendre transforms 396
14.3	Hamilton's equations 400
14.4	Hamiltonian phase space ((q, p) -space) 406
14.5	Liouville's theorem and recurrence 408
	Problems 413
4	Further topics 419
15	The general theory of small oscillations 421
15.1	Stable equilibrium and small oscillations 421
15.2	The approximate forms of T and V 425
15.3	The general theory of normal modes 429
15.4	Existence theory for normal modes 433
15.5	Some typical normal mode problems 436
15.6	Orthogonality of normal modes 444
15.7	General small oscillations 447
15.8	Normal coordinates 448
	Problems 452

Contents	ix
16 Vector angular velocity and rigid body kinematics	457
16.1 Rotation about a fixed axis	457
16.2 General rigid body kinematics	460
Problems	467
17 Rotating reference frames	469
17.1 Transformation formulae	469
17.2 Particle dynamics in a non-inertial frame	476
17.3 Motion relative to the Earth	478
17.4 Multi-particle system in a non-inertial frame	485
Problems	489
18 Tensor algebra and the inertia tensor	492
18.1 Orthogonal transformations	493
18.2 Rotated and reflected coordinate systems	495
18.3 Scalars, vectors and tensors	499
18.4 Tensor algebra	505
18.5 The inertia tensor	508
18.6 Principal axes of a symmetric tensor	514
18.7 Dynamical symmetry	516
Problems	519
19 Problems in rigid body dynamics	522
19.1 Equations of rigid body dynamics	522
19.2 Motion of ‘spheres’	524
19.3 The snooker ball	525
19.4 Free motion of bodies with axial symmetry	527
19.5 The spinning top	531
19.6 Lagrangian dynamics of the top	535
19.7 The gyrocompass	541
19.8 Euler’s equations	544
19.9 Free motion of an unsymmetrical body	549
19.10 The rolling wheel	556
Problems	560
Appendix Centres of mass and moments of inertia	564
A.1 Centre of mass	564
A.2 Moment of inertia	567
A.3 Parallel and perpendicular axes	571
Answers to the problems	576
<i>Bibliography</i>	589
<i>Index</i>	591