

## Index

A page number printed in **bold** indicates that a definition or an important statement about the index entry appears on that page.

- acceleration
  - in polar coordinates, 34
  - rectilinear, **26**
  - vector, **28**
- action functional, **384**
- angular momentum, 289–306
  - about a point, **290**
  - about an axis, **291**
  - conservation of, 298–306
    - about a point, **298**
    - about an axis, **299**
  - in planar motion, 307
  - of a rigid body, 292–293
    - about its rotation axis, **293**
    - in general motion, **510**
- angular momentum principle, 294–297
  - about a fixed point, **295**
  - about the centre of mass, **296**
  - non-standard, 317
- angular velocity
  - scalar, **37**, 36–40
  - vector, **457**, 457–464
    - existence of, 461–463
    - transformation of, 508
- apse, **162**
- apsidal angle, **162**
- apsidal distance, **162**, 163
- astronomical units, 174
- autonomous system(s), **200**, 353, 404
  - finding phase paths of, 200
  - phase paths of, 408
  - properties of, 201–202
- axis
  - notation for, 572
- basis set, 6–8
  - handedness of, 7
  - orthonormal, 7
  - standard, **8**
- Bertrand's theorem, 169
- bicycle
  - stability of, 559
- binary star, 268
  - period of, **267**
- brachistochrone problem, 378
- Brahe, Tycho*, 155
- calculus of variations, 369–380
  - with many variables, 385
- central field, **157**
  - angular momentum conservation in, 158
  - whose orbits are all closed, 168
- centre of mass, **58**, **564**, 564–567
  - equation, **247**
  - finding, by integration, 565–567
  - finding, by symmetry, 564
  - motion of, **248**, 247–250
  - of a hemisphere, 566
  - of a semi-circular lamina, 565
- centrifugal force, 477
- chaos
  - in Duffing's equation, 213, 218
  - not in integrable Hamiltonian systems, 417
  - not in plane autonomous systems, 210
- charged particle
  - Hamiltonian of, 414
  - in a magnetic field, 96
  - Lagrangian of, 351, 363
- circular motion, 92–98
  - non-uniform, 35
  - uniform, 30
- collision process, 255–264
  - elastic, **257**, 257–259
  - elastic collision formulae, **263**
  - energy conserved in, 258
  - energy principle for, 257
  - in the ZM frame, 259–261
  - linear momentum conserved in, **257**
  - meaning of, 255
- collision theory, *see* collision process

- configuration  
 of a system, **221**, 325  
 configuration space, 330–333  
 conics  
 geometry of, 184–186  
 conservation principles  
 and symmetry, 356–361  
 constraint force(s), 78, 325  
 that do no work, 146, 228  
 constraint(s), 324–325  
 examples of, 324  
 geometrical, 324  
 integrable, 328  
 kinematical, 324, 328–330  
 moving, 344–348  
 examples of, 344  
 Lagrange’s equations for, 345  
 non-integrable, 329  
 Coriolis force, 477  
*Coriolis, Gaspard-Gustav*, 475  
 couple, **287**  
 cyclic coordinate, 355, 406
- d’Alembert’s principle, **335**, 333–335  
*d’Alembert, Jean le Rond*, 334  
 degrees of freedom, **222**, **327**,  
 327–328  
 directed line segment, 4  
 drag, *see* resistance  
 drag coefficient, 83  
 Duffing’s equation  
 driven, 211  
 chaotic solutions of, 218  
 sub-harmonic resonance of, 213  
 undriven, 195  
 dynamical balancing, 548  
 dynamical symmetry, 516–518  
 axial, 517  
 none, 517  
 spherical, 518
- elastic collision formulae, **263**  
 embedded frame, *see* Euler’s equations  
 energy conservation  
 for a constrained system, **229**,  
 228–233  
 for a particle  
 in 3-D motion, **143**  
 in a conservative field, 140–143  
 in constrained motion, **146**, 145–146  
 in rectilinear motion, **134**, 133–134  
 for an unconstrained system, **227**, 225–228  
 energy function, **352**  
 conservation of, 353  
 for a conservative system, 354  
 energy principle  
 for a particle, **132**  
 for a system, **224**, 223–224
- equilibrium  
 of a conservative system, 421–424  
 condition for, **422**  
 stable, **423**  
 condition for, **423**  
 equilibrium position  
 of a particle, 138–139  
 oscillations about, 139  
 stability of, **139**  
 equivalence, principle of, **68**  
 escape speed, 78  
 Euler’s angles, 535  
 Euler’s equations, 544–549  
 deficiencies of, 548  
 Euler–Lagrange equation(s), **374**  
 integral of, 377  
 with many variables, **386**  
 extremals, **374**
- Fermat’s principle, **381**, 380–382  
 field, *see* force, field of  
 force  
 definition of, **56**  
 external, 223  
 fictitious, 477–478  
 centrifugal, 477  
 Coriolis, 477  
 field of, **74**  
 conservative, **141**  
 internal, 223  
 conservative, 226  
 units of, 51  
 Foucault pendulum, 482–484  
*Foucault, Jean Bernard Leon*, 484  
 Fourier series, 117  
 Fourier’s theorem, **117**  
*Fourier, Jean Baptiste Joseph*, 117  
 frames  
 in relative motion, 40  
 inertial, **53**, 52–54  
 mutually unaccelerated, **43**  
 rotating, *see* rotating frame(s)  
 frisbee  
 wobbling, 530  
 functional  
 extremal of, **374**  
 local minimum of, 372  
 made stationary, **372**, **385**  
 meaning of, 369
- generalised coordinates, 222, **326**, 325–330  
 generalised eigenvalues, **433**  
 of a symmetric matrix, **433**  
 generalised eigenvectors, **433**  
 orthogonality of, 444  
 generalised force, **337**  
 generalised momenta, **354**, 354–356  
 conservation of, **355**, **406**

- generalised velocities, **331**, 330–332  
 geodesic, 367  
 grad operator  
    $n$ -dimensional, 399  
   3-dimensional, 141  
 gravitation  
   by a distribution of mass, 60–67  
   by spheres, 64–67  
   constant of, 60  
   law of, **59**  
 gravitational acceleration, 68  
 gyrocompass, 541–544
- Hamilton form, 395, 396  
 Hamilton's equations, **402**, 400–403  
   in Poisson bracket form, 417  
 Hamilton's principle, **386**, 383–388  
   significance of, 387  
   with one degree of freedom, 383  
*Hamilton, Sir William Rowan*, 367, 402  
 Hamiltonian function, **401**  
   conservation of, 404  
   equation defining, **401**  
   explicit time dependence in, 402  
   for a conservative system, 404  
 Hohmann transfer, 177–178  
   optimality of, 186–188  
 holonomic system, **330**  
 Hooke's law, **106**  
*Hooke, Robert*, 106
- inertia tensor, **510**, 508–516  
   elements of, **511**  
   principal axes of, **515**, 514–516  
   principal moments in, **515**  
 inertia, law of, **53**  
 inertial frame, *see* frame, inertial  
 integrable system(s), 302–306  
 integrable systems, **273**, 273–277  
 invariance  
   of potential energy  
     under rotation, 358  
     under translation, 356  
   of the Lagrangian, 359  
 inverse square field (attractive), 170–178  
   E-formula for, **172**  
   Kepler's equation for, **176**  
   L-formula for, **171**  
   orbits in, 170  
   period formula for, **174**  
   period of elliptic orbit in, 173  
   time dependence of motion in, 175  
 inverse square field (repulsive), 179  
   E-formula for, **179**  
   L-formula for, **179**  
   orbits in, 179  
 isolated system, 250, 298
- Kepler's laws, **156**  
*Kepler, Johannes*, 155  
 kinetic energy, **132**  
   of a rigid body  
     in general motion, **238**, 237–238  
     in general rotational motion, **510**  
     with a fixed axis, **235**, 234–235
- laboratory (lab) frame, **259**  
 Lagrange's equations, 335–343  
   for a conservative system, **339**  
   for a general system, **338**  
   in Lagrangian form, **349**  
   partial derivatives in, 336  
   sufficiency of, 339  
   summary of method, 339  
   with moving constraints, **346**  
*Lagrange, Joseph-Louis*, 323  
 Lagrangian, **349**, 348–351  
   invariance of, 359  
 Legendre transform(s), **400**, 396–400  
   active and passive variables in, 399  
   with many variables, 399  
   with two variables, 396  
 limit cycle(s), 205–210  
   mechanical example of, 209  
   not in Hamiltonian dynamics, 410  
   of Rayleigh's equation, 207  
   of Van der Pol's equation, 217  
   proving existence of, 206–209  
 line integral, 140  
 linear momentum  
   component, conservation of, **251**  
   conservation of, **250**, 250–251  
   of a particle, **245**  
   of a system, **245**  
 linear momentum principle, **247**  
 Linstedt's method, 197–199  
 Liouville's theorem, **409**  
   on integrable systems, 417  
   use in statistical mechanics, 415  
*Liouville, Joseph*, 409, 417  
*Littlewood, John Edensor*, 478  
 Lorentz force, **350**
- mass  
   gravitational, 67  
   inertial, **55**  
   units of, 51  
 Maxwell's equations  
   in dynamic case, 363  
   in static case, 350  
 molecular vibrations, 438–440  
 moment of a force, 286–289  
   about a point, **286**  
   about an axis, **288**  
   geometrical meaning of, 287

- moment of inertia, **234**, **567**, 567–575  
 of a circular disk, 568, 570, 574  
 of a cylinder, 570  
 of a hoop, 570  
 of a rectangular block, 574  
 of a rectangular plate, 573, 574  
 of a rod, 568, 570  
 of a solid sphere, 569, 570  
 of a spherical shell, 570  
 parallel axes theorem for, 571  
 perpendicular axes theorem for, 573  
 table of, **570**
- momentum, *see* linear momentum  
 or angular momentum  
 or generalised momentum
- multiple interactions, law of, **57**
- mutual interaction, law of, **54**
- Newton, Sir Isaac*, 50
- Newton's Laws, **51**  
 experimental basis of, 58
- Noether's theorem, **359**, 358–361
- Noether, Emmy Amalie*, 358
- non-linear oscillations  
 driven, 211–214  
 free periodic, 194–199  
 Linstedt's method, 197–199  
 proving existence of, 195  
 unsymmetrical, 215
- normal coordinates, **448**, 448–452  
 determination of, **451**
- normal frequency, **123**, **431**  
 degenerate, **435**  
 determinantal equation for, **430**
- normal mode(s), **122**, 121–126, **429**  
 amplitude vector of, **430**  
 existence of, 433–435, **435**  
 orthogonality of, 444–445  
 orthogonality relations for, **445**  
 orthonormality relations for, **445**  
 Rayleigh's principle for, 445  
*T*-matrix for, **428**  
 typical problems on, 436–444  
*V*-matrix for, **428**
- normal vector (to a curve), 20–22
- nutation, 538
- ODEs  
 first order systems of, 200, 393
- one-body problem, 157
- orbit  
 aphelion of, **162**  
 apogee of, **162**  
 perigee of, **162**  
 perihelion of, **162**
- orbital motion  
 angular momentum equation for, **159**  
 effective potential in, 161  
 energy equation for, **160**  
 general nature of, 159–164  
 nearly circular, 167–169  
 path equation for, **165**, 164–167  
 radial motion equation for, **160**  
 stability of, 168
- orthogonal diagonalisation  
 of a symmetric matrix, **514**
- orthogonal matrix, **494**
- orthogonal transformation, **495**,  
 493–499  
 determinant of, 499
- oscillations (linear)  
 coupled, 121–126  
 driven, **107**  
 free, **107**
- particle, 51
- perturbation expansion  
 Linstedt's method, 197–199  
 regular, **196**
- phase diagram, 200  
 for damped SHO, 203
- phase fluid, 408
- phase path(s), 200  
 closed, 202  
 in polar form, 215
- phase space, 199–211  
 (*q*, *p*)-space, 406–408  
 (*x*, *v*)-plane, 202  
 (*x*<sub>1</sub>, *x*<sub>2</sub>)-plane, 200
- planar motion, **306**, 306–313  
 of a rigid body, 308  
 rigid body equations for, 308
- Poincaré–Bendixson theorem, **205**  
 applications of, 206–209, 217
- Poisson brackets, 416  
 Jacobi's identity for, 416
- polar coordinates, **32**
- polar unit vectors, **32**
- pool ball, *see* snooker ball
- position vector, **8**
- potential  
 electrostatic, 350  
 magnetic vector, 350  
 velocity dependent, 349–351
- potential energy  
 mutual, 226  
 of a particle  
 in 3-D motion, **141**  
 in rectilinear motion, **134**
- of a system  
 external, 225  
 internal, 225–227  
 total, 227  
 under uniform gravity, 225
- precession  
 of a top, 532–534, 539–541  
 of planetary orbits, 169
- predator-prey equations, 200, 394
- periodicity of solutions, 216

- product(s) of inertia, 511  
 products of vectors, *see* vectors  
 projectile motion, 88–92  
   with resistance, 90–92  
 pseudovector(s), **508**
- recurrence theorem, **411**  
   stronger form of, **412**  
 reduced mass, **266**  
 reference frame, *see* frame  
 resistance  
   linear law of, 83  
   physics of, 82  
   quadratic law of, 83  
 resisted motion, 82–87  
 resonance, **116**, 115–116  
   sub-harmonic, 213  
   super-harmonic, 213, 217  
 resonant frequency, **116**  
 response  
   driven, **113**  
   transient, **113**  
 Reynolds number, **83**  
 rigid body (dynamics of)  
   axially symmetric, 527–544  
   under no forces, **530**, 527–531  
   equations of motion for  
     in general motion, **522**  
     with a fixed point, **523**  
   in planar motion, 306–313  
   spherically symmetric, 524–527  
   unsymmetrical, 549–556  
     free motion of, 551–556  
     stability of rotation of, 553  
 rigid body (kinematics of)  
   in general motion, 460–467  
     angular velocity of, **461**, **462**  
   planar motion of, **38**, 38–40  
   with a fixed axis, 36, 457–460  
   particle velocities in, **459**  
 rigid body equations, **297**  
 rigidity, **457**  
   conditions for, 460  
 rocket equation  
   in free space, **253**  
   under gravity, **254**  
 rocket motion, 251–255  
 rolling condition, **40**, 325, 464  
 rolling wheel, 556–559  
   stability of, 558  
 ropes  
   discrete representation of, 231  
 Rossby number, **484**  
 Rossby, Carl-Gustav, 484  
 rotating Earth, 478–485  
   apparent value of  $g$  on, **479**  
   apparent vertical on, **479**  
   geostrophic flow on, 485  
   pendulum on, 482–484  
   projectile equation for, **480**  
   projectile motion on, 480  
   wind on, 484–485  
 rotating frame(s)  
   apparent acceleration in, **475**  
   apparent angular velocity in, **474**  
   apparent velocity in, **473**  
   basis vectors in, 470  
     true rates of change of, **472**  
   energy conservation in, **487**  
   energy principle in, 486–488  
   form of Second Law in, **476**  
   linear momentum principle in, 485  
   true and apparent values in, 471  
 Rutherford, Sir Ernest, 179  
 Rutherford scattering, 179–184  
   one-body formula, **184**  
   two-body formula, **272**
- SI units, 51  
 scalar (invariant), **500**  
 scattering cross section, 181  
   formula in axisymmetric case, 183  
   general relation satisfied by, 182  
 seismograph, 120  
 SHM, *see* simple harmonic motion  
 SHM equation, **107**  
   with damping, **109**  
   with damping and driven, **112**  
 simple harmonic motion  
   driven, 112–119  
   driven by harmonic force, 112–115  
   driven by periodic force, 117–119  
   free, critically damped, 127  
   free, damped, 109–111  
   free, overdamped, 111  
   free, undamped, 107–109  
   free, underdamped, 110–111  
 simple pendulum, 93–96  
   exact equation for, **94**  
   exact period of, 95  
   linear theory of, 94  
 Snell's law, 380  
 snooker ball  
   dynamics of, 525–527  
   putting 'side' on, 465  
   rolling of, 464  
 speed, 29  
 spherical pendulum, 302  
 spinning top  
   Lagrangian for, **536**  
   solved analytically, 535–541  
   solved vectorially, 531–534  
 spring  
   linear, **106**  
     spring constant (strength) of, **106**  
   non-linear  
     hardening, **194**  
     softening, **194**  
   restoring force of, 106  
 standard mechanical system, **335**

- statics
  - of rigid bodies, *313–316*
  - equations for, **314**
- Stokes drag formula, *83*
- Stokes, George Gabriel*, *83*
- string
  - inextensible, *80*
  - tension in, *80*
- summation convention, *502*
- symmetry
  - and conservation principles, *356–361*
- system(s)
  - constrained, *221*
  - unconstrained, *221*
  
- T*-matrix, **428**
- tangent vector (to a curve), *18–20*
- tensor(s)
  - contraction of, *506*
  - identity, **505**
  - isotropic, **505**
  - outer product of, *505*
  - second order, **501**
    - in matrix form, **503**
  - summation convention for, *502*
  - third order, **502**
  - vector regarded as, **501**
- three-body problem, *417*
- top, *see* spinning top
- torque, *286*
- transformation matrix, **494**
  - of a given reflection, *498–499*
  - of a given rotation, *495–498*
- two-body problem, *264–269*
  - reduced mass in, **266**
  - relative motion equation, **266**
  - relative motion in, **266**
- two-body scattering, *269–273*
  - cross section formula for, **271**
  - in Rutherford case, **272**
  - in Rutherford case, **270**
  
- V*-matrix, **428**
- variation, *371*
  - admissible, *371*
  - norm of, *371*
- variational principles, *380–388*
- vector algebra, *3–16*
- vector calculus, *16–18*
- vector geometry, *8–10*
- vector quantity, **3**
- vector(s), **4**
  - as first order tensors, **501**
  - basis set of, *6–8*
  - components of, **12, 12–13**
  - difference of, **5**
  - differentiation of, **16, 16–18**
  - scalar multiple of, **5, 5–6**
  - scalar product of, **10, 10–12**
  - sum of, **5, 5**
  - triple scalar product of, *15*
  - triple vector product of, *15–16*
  - unit, *6*
  - vector product of, **13, 13–14**
- velocity
  - circumferential, **35**
  - in polar coordinates, *34*
  - rectilinear, **26**
  - vector, **28**
- velocity diagram(s), **273, 273–274**
- virtual motion, **334**
- virtual work, *334*
  - principle of, **335**
  
- weight, *69*
- work
  - done by a force, **132, 133, 141**
  - SI unit of, **132**
  
- zero momentum (ZM) frame, **260**