

## CLASSICAL MECHANICS

Gregory's *Classical Mechanics* is a major new textbook for undergraduates in mathematics and physics. It is a thorough, self-contained and highly readable account of a subject many students find difficult. The author's clear and systematic style promotes a good understanding of the subject: each concept is motivated and illustrated by worked examples, while problem sets provide plenty of practice for understanding and technique. Computer assisted problems, some suitable for projects, are also included. The book is structured to make learning the subject easy; there is a natural progression from core topics to more advanced ones and hard topics are treated with particular care. A theme of the book is the importance of conservation principles. These appear first in vectorial mechanics where they are proved and applied to problem solving. They reappear in analytical mechanics, where they are shown to be related to symmetries of the Lagrangian, culminating in Noether's theorem.

- Suitable for a wide range of undergraduate mechanics courses given in mathematics and physics departments: no prior knowledge of the subject is assumed
- Profusely illustrated and thoroughly class-tested, with a clear direct style that makes the subject easy to understand: all concepts are motivated and illustrated by the many worked examples included
- Good, accurately-set problems, with answers in the book: computer assisted problems and projects are also provided. Model solutions for problems available to teachers from [www.cambridge.org/Gregory](http://www.cambridge.org/Gregory)

### The author

Douglas Gregory is Professor of Mathematics at the University of Manchester. He is a researcher of international standing in the field of elasticity, and has held visiting positions at New York University, the University of British Columbia, and the University of Washington. He is highly regarded as a teacher of applied mathematics: this, his first book, is the product of many years of teaching experience.

Cambridge University Press  
978-0-521-82678-5 - Classical Mechanics: An Undergraduate Text  
R. Douglas Gregory  
Frontmatter  
[More information](#)

---

*Bloody instructions, which, being taught,  
Return to plague th' inventor.*

SHAKESPEARE, *Macbeth*, act I, sc. 7

**Front Cover** The photograph on the front cover shows Mimas, one of the many moons of Saturn; the huge crater was formed by an impact. Mimas takes 22 hours 37 minutes to orbit Saturn, the radius of its orbit being 185,500 kilometres. After reading Chapter 7, you will be able to estimate the mass of Saturn from this data!

Cambridge University Press  
978-0-521-82678-5 - Classical Mechanics: An Undergraduate Text  
R. Douglas Gregory  
Frontmatter  
[More information](#)

---

# CLASSICAL MECHANICS

AN UNDERGRADUATE TEXT

---

R. DOUGLAS GREGORY

*University of Manchester*



CAMBRIDGE  
UNIVERSITY PRESS

Cambridge University Press  
978-0-521-82678-5 - Classical Mechanics: An Undergraduate Text  
R . Douglas Gregory  
Frontmatter  
[More information](#)

---

**CAMBRIDGE**  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.  
It furthers the University's mission by disseminating knowledge in the pursuit of  
education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521826785](http://www.cambridge.org/9780521826785)

© Cambridge University Press 2006

This publication is in copyright. Subject to statutory exception  
and to the provisions of relevant collective licensing agreements,  
no reproduction of any part may take place without  
the written permission of Cambridge University Press.

First published 2006  
11th printing 2015

Printed in the United Kingdom by Clays, St Ives plc.

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-82678-5 Hardback  
ISBN 978-0-521-53409-3 Paperback

---

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party  
internet websites referred to in this publication, and does not guarantee that any content on such websites is,  
or will remain, accurate or appropriate.

---

# Contents

<i>Preface</i> . . . . .	xi
<b>1 Newtonian mechanics of a single particle</b>	<b>1</b>
<b>1 The algebra and calculus of vectors</b>	<b>3</b>
1.1 Vectors and vector quantities . . . . .	3
1.2 Linear operations: $\mathbf{a} + \mathbf{b}$ and $\lambda \mathbf{a}$ . . . . .	5
1.3 The scalar product $\mathbf{a} \cdot \mathbf{b}$ . . . . .	10
1.4 The vector product $\mathbf{a} \times \mathbf{b}$ . . . . .	13
1.5 Triple products . . . . .	15
1.6 Vector functions of a scalar variable . . . . .	16
1.7 Tangent and normal vectors to a curve . . . . .	18
<b>Problems</b> . . . . .	22
<b>2 Velocity, acceleration and scalar angular velocity</b>	<b>25</b>
2.1 Straight line motion of a particle . . . . .	25
2.2 General motion of a particle . . . . .	28
2.3 Particle motion in polar co-ordinates . . . . .	32
2.4 Rigid body rotating about a fixed axis . . . . .	36
2.5 Rigid body in planar motion . . . . .	38
2.6 Reference frames in relative motion . . . . .	40
<b>Problems</b> . . . . .	43
<b>3 Newton's laws of motion and the law of gravitation</b>	<b>50</b>
3.1 Newton's laws of motion . . . . .	50
3.2 Inertial frames and the law of inertia . . . . .	52
3.3 The law of mutual interaction; mass and force . . . . .	54
3.4 The law of multiple interactions . . . . .	57
3.5 Centre of mass . . . . .	58

vi	Contents
3.6	The law of gravitation . . . . . 59
3.7	Gravitation by a distribution of mass . . . . . 60
3.8	The principle of equivalence and $g$ . . . . . 67
	<b>Problems</b> . . . . . 71
<b>4</b>	<b>Problems in particle dynamics</b> <b>73</b>
4.1	Rectilinear motion in a force field . . . . . 74
4.2	Constrained rectilinear motion . . . . . 78
4.3	Motion through a resisting medium . . . . . 82
4.4	Projectiles . . . . . 88
4.5	Circular motion . . . . . 92
	<b>Problems</b> . . . . . 98
<b>5</b>	<b>Linear oscillations</b> <b>105</b>
5.1	Body on a spring . . . . . 105
5.2	Classical simple harmonic motion . . . . . 107
5.3	Damped simple harmonic motion . . . . . 109
5.4	Driven (forced) motion . . . . . 112
5.5	A simple seismograph . . . . . 120
5.6	Coupled oscillations and normal modes . . . . . 121
	<b>Problems</b> . . . . . 126
<b>6</b>	<b>Energy conservation</b> <b>131</b>
6.1	The energy principle . . . . . 131
6.2	Energy conservation in rectilinear motion . . . . . 133
6.3	General features of rectilinear motion . . . . . 136
6.4	Energy conservation in a conservative field . . . . . 140
6.5	Energy conservation in constrained motion . . . . . 145
	<b>Problems</b> . . . . . 151
<b>7</b>	<b>Orbits in a central field</b> <b>155</b>
7.1	The one-body problem – Newton’s equations . . . . . 157
7.2	General nature of orbital motion . . . . . 159
7.3	The path equation . . . . . 164
7.4	Nearly circular orbits . . . . . 167
7.5	The attractive inverse square field . . . . . 170
7.6	Space travel – Hohmann transfer orbits . . . . . 177
7.7	The repulsive inverse square field . . . . . 179
7.8	Rutherford scattering . . . . . 179
Appendix A	The geometry of conics . . . . . 184
Appendix B	The Hohmann orbit is optimal . . . . . 186
	<b>Problems</b> . . . . . 188

<b>Contents</b>	vii
<b>8 Non-linear oscillations and phase space</b>	<b>194</b>
8.1 Periodic non-linear oscillations . . . . .	194
8.2 The phase plane $((x_1, x_2)$ -plane) . . . . .	199
8.3 The phase plane in dynamics $((x, v)$ -plane) . . . . .	202
8.4 Poincaré-Bendixson theorem: limit cycles . . . . .	205
8.5 Driven non-linear oscillations . . . . .	211
<b>Problems</b> . . . . .	214
<b>2 Multi-particle systems</b>	<b>219</b>
<b>9 The energy principle</b>	<b>221</b>
9.1 Configurations and degrees of freedom . . . . .	221
9.2 The energy principle for a system . . . . .	223
9.3 Energy conservation for a system . . . . .	225
9.4 Kinetic energy of a rigid body . . . . .	233
<b>Problems</b> . . . . .	241
<b>10 The linear momentum principle</b>	<b>245</b>
10.1 Linear momentum . . . . .	245
10.2 The linear momentum principle . . . . .	246
10.3 Motion of the centre of mass . . . . .	247
10.4 Conservation of linear momentum . . . . .	250
10.5 Rocket motion . . . . .	251
10.6 Collision theory . . . . .	255
10.7 Collision processes in the zero-momentum frame . . . . .	259
10.8 The two-body problem . . . . .	264
10.9 Two-body scattering . . . . .	269
10.10 Integrable mechanical systems . . . . .	273
Appendix A Modelling bodies by particles . . . . .	277
<b>Problems</b> . . . . .	279
<b>11 The angular momentum principle</b>	<b>286</b>
11.1 The moment of a force . . . . .	286
11.2 Angular momentum . . . . .	289
11.3 Angular momentum of a rigid body . . . . .	292
11.4 The angular momentum principle . . . . .	294
11.5 Conservation of angular momentum . . . . .	298
11.6 Planar rigid body motion . . . . .	306
11.7 Rigid body statics in three dimensions . . . . .	313
<b>Problems</b> . . . . .	317

<b>3</b>	<b>Analytical mechanics</b>	<b>321</b>
<b>12</b>	<b>Lagrange's equations and conservation principles</b>	<b>323</b>
12.1	Constraints and constraint forces . . . . .	323
12.2	Generalised coordinates . . . . .	325
12.3	Configuration space ( $q$ -space) . . . . .	330
12.4	D'Alembert's principle . . . . .	333
12.5	Lagrange's equations . . . . .	335
12.6	Systems with moving constraints . . . . .	344
12.7	The Lagrangian . . . . .	348
12.8	The energy function $h$ . . . . .	351
12.9	Generalised momenta . . . . .	354
12.10	Symmetry and conservation principles . . . . .	356
	<b>Problems</b> . . . . .	361
<b>13</b>	<b>The calculus of variations and Hamilton's principle</b>	<b>366</b>
13.1	Some typical minimisation problems . . . . .	367
13.2	The Euler–Lagrange equation . . . . .	369
13.3	Variational principles . . . . .	380
13.4	Hamilton's principle . . . . .	383
	<b>Problems</b> . . . . .	388
<b>14</b>	<b>Hamilton's equations and phase space</b>	<b>393</b>
14.1	Systems of first order ODEs . . . . .	393
14.2	Legendre transforms . . . . .	396
14.3	Hamilton's equations . . . . .	400
14.4	Hamiltonian phase space ( $(q, p)$ -space) . . . . .	406
14.5	Liouville's theorem and recurrence . . . . .	408
	<b>Problems</b> . . . . .	413
<b>4</b>	<b>Further topics</b>	<b>419</b>
<b>15</b>	<b>The general theory of small oscillations</b>	<b>421</b>
15.1	Stable equilibrium and small oscillations . . . . .	421
15.2	The approximate forms of $T$ and $V$ . . . . .	425
15.3	The general theory of normal modes . . . . .	429
15.4	Existence theory for normal modes . . . . .	433
15.5	Some typical normal mode problems . . . . .	436
15.6	Orthogonality of normal modes . . . . .	444
15.7	General small oscillations . . . . .	447
15.8	Normal coordinates . . . . .	448
	<b>Problems</b> . . . . .	452



<b>Contents</b>	<b>ix</b>
<b>16 Vector angular velocity and rigid body kinematics</b>	<b>457</b>
16.1 Rotation about a fixed axis . . . . .	457
16.2 General rigid body kinematics . . . . .	460
<b>Problems</b> . . . . .	467
<b>17 Rotating reference frames</b>	<b>469</b>
17.1 Transformation formulae . . . . .	469
17.2 Particle dynamics in a non-inertial frame . . . . .	476
17.3 Motion relative to the Earth . . . . .	478
17.4 Multi-particle system in a non-inertial frame . . . . .	485
<b>Problems</b> . . . . .	489
<b>18 Tensor algebra and the inertia tensor</b>	<b>492</b>
18.1 Orthogonal transformations . . . . .	493
18.2 Rotated and reflected coordinate systems . . . . .	495
18.3 Scalars, vectors and tensors . . . . .	499
18.4 Tensor algebra . . . . .	505
18.5 The inertia tensor . . . . .	508
18.6 Principal axes of a symmetric tensor . . . . .	514
18.7 Dynamical symmetry . . . . .	516
<b>Problems</b> . . . . .	519
<b>19 Problems in rigid body dynamics</b>	<b>522</b>
19.1 Equations of rigid body dynamics . . . . .	522
19.2 Motion of ‘spheres’ . . . . .	524
19.3 The snooker ball . . . . .	525
19.4 Free motion of bodies with axial symmetry . . . . .	527
19.5 The spinning top . . . . .	531
19.6 Lagrangian dynamics of the top . . . . .	535
19.7 The gyrocompass . . . . .	541
19.8 Euler’s equations . . . . .	544
19.9 Free motion of an unsymmetrical body . . . . .	549
19.10 The rolling wheel . . . . .	556
<b>Problems</b> . . . . .	560
<b>Appendix Centres of mass and moments of inertia</b>	<b>564</b>
A.1 Centre of mass . . . . .	564
A.2 Moment of inertia . . . . .	567
A.3 Parallel and perpendicular axes . . . . .	571
<b>Answers to the problems</b>	<b>576</b>
<i>Bibliography</i> . . . . .	589
<i>Index</i> . . . . .	591

Cambridge University Press  
978-0-521-82678-5 - Classical Mechanics: An Undergraduate Text  
R . Douglas Gregory  
Frontmatter  
[More information](#)

---

# Preface

---

## Information for readers

### What is this book about and who is it for?

This is a book on **classical mechanics** for **university undergraduates**. It aims to cover all the material normally taught in classical mechanics courses from Newton's laws to Hamilton's equations. If you are attending such a course, you will be unlucky not to find the course material in this book.

### What prerequisites are needed to read this book?

It is expected that the reader will have attended an elementary **calculus** course and an elementary course on **differential equations** (ODEs). A previous course in mechanics is helpful but not essential. *This book is self-contained in the sense that it starts from the beginning and assumes no prior knowledge of mechanics.* However, in a general text such as this, the early material is presented at a brisker pace than in books that are specifically aimed at the beginner.

### What is the style of the book?

The book is written in a crisp, no nonsense style; in short, there is no waffle! The object is to get the reader to the important points as quickly and easily as possible, consistent with good understanding.

### Are there plenty of examples with full solutions?

Yes there are. Every new concept and technique is reinforced by **fully worked examples**. The author's advice is that the reader should think how he or she would do each worked example *before* reading the solution; much more will be learned this way!

### Are there plenty of problems with answers?

Yes there are. At the end of each chapter there is a large collection of problems. For convenience, these are arranged by topic and trickier problems are marked with a star. **Answers are provided to all of the problems.** A feature of the book is the inclusion of computer assisted problems. These are interesting physical problems that cannot be solved analytically, but can be solved easily with computer assistance.

### Where can I find more information?

More information about this book can be found on the book's homepage

<http://www.cambridge.org/Gregory>

All feedback from readers is welcomed. Please e-mail your comments, corrections and good ideas by clicking on the comments button on the book's homepage.

## Information for lecturers

### Scope of the book and prerequisites

This book aims to cover all the material normally taught in undergraduate mechanics courses from Newton's laws to Hamilton's equations. It assumes that the students have attended an elementary calculus course and an elementary course on ODEs, but no more. The book is self contained and, in principle, it is not essential that the students should have studied mechanics before. However, their lives will be made easier if they have!

### Inspection copy and Solutions Manual

Any lecturer who is giving an undergraduate course on classical mechanics can request an **inspection copy** of this book. Simply go to the book's homepage

<http://www.cambridge.org/Gregory>

and follow the links.

Lecturers who adopt this book for their course may receive the **Solutions Manual**. This has a **complete set of detailed solutions** to the problems at the end of the chapters. To obtain the Solutions Manual, just send an e-mail giving your name, affiliation, and details of the course to [solutions@cambridge.org](mailto:solutions@cambridge.org)

### Feedback

All feedback from instructors and lecturers is welcomed. Please e-mail your comments via the link on the book's homepage

## Acknowledgements

I am very grateful to many friends and colleagues for their helpful comments and suggestions while this book was in preparation. But most of all I thank my wife Win for her unstinting support and encouragement, without which the book could not have been written at all.