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Logic, Induction and Sets

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Preface

When there are so many textbooks on logic already available, an author of a new one must expect to be challenged for explanations as to why he has added to their number. I have four main excuses. I am not happy with the treatments of well-foundedness nor of the axiomatisation of set theory in any of the standard texts known to me. My third excuse is that, because my first degree was not in mathematics but in philosophy and music, I have always been more preoccupied with philosophical concerns than have most of my colleagues. Both the intension-extension distinction and the use-mention distinction are not only philosophically important but pedagogically important too: this is no coincidence. Many topics in mathematics become much more accessible to students if approached in a philosophically sensitive way. My fourth excuse is that nobody has yet written an introductory book on logic that fully exploits the expository possibilities of the idea of an inductively defined set or recursive datatype. I think my determination to write such a book is one of the *sequelæ* of reading Conway's beautiful book (2001) based on lectures he gave in Cambridge many years ago when I was a Ph.D. student.

This book is based on my lecture notes and supervision (tutorial) notes for the course entitled "Logic, Computation and Set Theory", which is lectured in part II (third year) of the Cambridge Mathematics Tripos. The choice of material is not mine, but is laid down by the Mathematics Faculty Board having regard to what the students have learned in their first two years. Third-year mathematics students at Cambridge have learned a great deal of mathematics, as Cambridge is one of the few schools where it is possible for an undergraduate to do nothing but mathematics for three years; however, they have done no logic to speak of. Readers who know more logic and less mathematics than did the original audience for this material – and among mathematicians they may well be a majority outside these islands – may find the emphasis rather odd. The part IIb course, of which this is a component, is designed for strong mathematics students who wish to go further and who need some exposure to logic: it was never designed to produce logicians. This book was written to meet a specific need, and to those with that need I offer it in the hope that it can be of help. I offer it also in the hope that it will convey to mathematicians something of the flavour of the distinctive way logicians do mathematics.

Like all teachers, I owe a debt to my students. Any researcher needs students for the stimulating questions they ask, and those attempting to write textbooks will be grateful to their students for the way they push us to give clearer explanations than our unreflecting familiarity with elementary material normally generates. At times students' questions will provoke us into saying things we had not realised we knew. I am also grateful to my colleagues Peter Johnstone and Martin Hyland for exercises they provided.