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978-0-521-82565-8 - Random Dynamical Systems: Theory and Applications

Rabi Bhattacharya and Mukul Majumdar

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