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0521825385 - Graph Directed Markov Systems: Geometry and Dynamics of Limit Sets

R. Daniel Mauldin and Mariusz Urbanski

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