1 Introduction

1.1 Atomic astrophysics and spectroscopy

Spectroscopy is the science of light–matter interaction. It is one of the most powerful scientific tools for studying nature. Spectroscopy is dependent on, and therefore reveals, the inherent as well as the extrinsic properties of matter. Confining ourselves to the present context, it forms the link that connects astronomy with fundamental physics at atomic and molecular levels. In the broadest sense, spectroscopy explains all that we see. It underlies vision itself, such as the distinction between colours. It enables the study of matter and light through the wavelengths of radiation ('colours') emitted or absorbed uniquely by each element. Atomic astrophysics is atomic physics and plasma physics applied to astronomy, and it underpins astrophysical spectroscopy. Historically, astrophysical spectroscopy is older than modern astrophysics itself. One may recall Newton's experiments in the seventeenth century on the dispersion of sunlight by a prism into the natural rainbow colours as an identification of the visible band of radiation. More specifically, we may trace the beginning of astrophysical spectroscopy in the early nineteenth century to the discovery of dark lines in the solar spectrum by Wollaston in 1802 and Fraunhofer in 1815. The dark lines at discrete wavelengths arise from removal or absorption of energy by atoms or ions in the solar atmosphere. Fraunhofer observed hundreds of such features that we now associate with several constituent elements in the Sun, such as the *sodium D lines*.

Figure 1.1 shows the *Fraunhofer lines*. Fraunhofer himself did not associate the lines with specific elements; that had to await several other crucial developments, including laboratory experiments, and eventually quantum theory. He labelled the lines alphabetically, starting from A in the far red towards shorter wavelengths. It is instructive to revisit the proper identification of these historic lines. Going from right to left in Fig. 1.1, the first two lines A (7594 Å) and B (6867 Å) do not originate in the Sun but are due to absorption by oxygen in the terrestrial atmosphere. The line C at 6563 Å is due to absorption by hydrogen (the same transition in *emission* is a bright red line). The three lines A, B and C lie towards the red end of the visible spectrum. In the middle region of the spectrum are the two orange lines D1 and D2 (5896, 5890 Å, respectively) that are the characteristic 'doublet' lines of sodium (sodium lamps have an orange hue, owing to emission in the very same transitions). Towards the blue end we have the strong line E at 5270 Å, due to absorption by neutral iron, and another line, F (4861 Å), due to hydrogen. The molecular G band of CH lies around 4300 Å. Farther into the blue, there are the H and K lines (3968, 3934 Å, respectively) from singly ionized calcium, which are among the strongest absorption lines in the solar spectrum. Although the letters have no physical meaning, this historic notation is carried through to the present day. Much of early astrophysics consisted of the identification of spectral lines, according to the presence of various atomic species in stars and nebulae.

The lightest and most abundant element in the Universe is hydrogen, chemical symbol H. The abundances and line intensities of other elements are expressed relative to H, which has the most common spectroscopic features in most astronomical sources. Observed line wavelengths led to an early grasp of specific spectra, but it needed the advent of quantum mechanics to understand the underlying structure. The pioneering exploration of the hydrogen spectrum and of alkali atoms by Rydberg was the first systematic attempt to analyze the pattern of spectral lines. We shall see later how useful simple variants of the empirical Rydberg formula can be in the analysis of astrophysical spectra.

Spectroscopy also predates quantum mechanics. In spite of the empirical work and analysis, a quantitative understanding of spectroscopy had to await the quantum

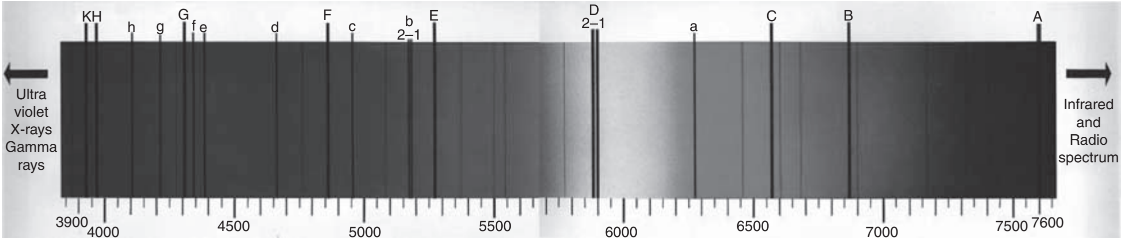

FIGURE 1.1 The Fraunhofer lines (Courtesy *Institute for Astronomy*, University of Hawaii, www.harmsy.freeuk.com).

theory of the atom. Schrödinger's success in finding the right equation that would reproduce the observed hydrogenic energy levels according to the Bohr model and the Rydberg formula was the crucial development. Mathematically, Schrödinger's equation is a rather straightforward second-order differential equation, well-known in mathematical analysis as Whittaker's equation [1]. It was the connection of its eigenvalues with the energy levels of the hydrogen atom that established basic quantum theory. In the next chapter, we shall retrace the derivation that leads to the quantization of apparently continuous variables such as energy. However, with the exception of the hydrogen atom, the main problem was (and to a significant extent still is!) that atomic physics gets complicated very fast as soon as one moves on to non-hydrogenic systems, starting with the very next element, helium. This is not unexpected, since only hydrogen (or the hydrogenic system) is a two-body problem amenable to an exact mathematical solution. All others are three-body or many-body problems that mainly have numerical solutions obtained on solving generalized forms of the Schrödinger's equation. With modern-day supercomputers, however, non-hydrogenic systems, particularly those of interest in astronomy, are being studied with increasing accuracy. A discussion of the methods and results is one of the main topics of this book.

Nearly all astronomy papers in the literature identify atomic transitions by wavelengths, and not by the spectral states involved in the transitions. The reason for neglecting basic spectroscopic information is because it is thought to be either too tedious or irrelevant to empirical analysis of spectra. Neither is quite true. But whereas the lines of hydrogen are well-known from undergraduate quantum mechanics, lines of more complicated species require more detailed knowledge. Strict rules, most notably the Pauli exclusion principle, govern the formation of atomic states. But their application is not straightforward, and the full algebraic scheme must be followed, in order to derive and understand which states are allowed by nature to exist, and which are not. Moreover, spectroscopic information for a given atom can be immensely valuable in correlating with other similar atomic species.

While we shall explore atomic structure in detail in the next chapter, even a brief historical sketch of atomic astrophysics would be incomplete without the noteworthy connection to stellar spectroscopy. In a classic paper in 1925 [2], Russell and Saunders implemented the then new science of quantum mechanics, in one of its first major applications, to derive the algebraic rules for recoupling total spin and angular momenta S and L of all electrons in an atom. The so-called *Russell–Saunders coupling* or *LS coupling* scheme thereby laid the basis for spectral identification of the states of an atom – and hence the foundation of much of atomic physics itself. Hertzsprung and Russell then went on to develop an extremely useful phenomenological description of stellar spectra based on spectral type (defined by atomic lines) vs. temperature or colour. The so-called *Hertzsprung–Russell (HR) diagram* that plots luminosity versus spectral type or temperature is the starting point for the classification of all stars (Chapter 10).

In this introductory chapter, we lay out certain salient properties and features of astrophysical sources.

1.2 Chemical and physical properties of elements

There are similarities and distinctions between the *chemical* and the *physical* properties of elements in the periodic table (Appendix 1). Both are based on the electronic arrangements in shells in atoms, divided in *rows* with increasing atomic number Z. The electrons, with principal quantum number n and orbital angular momentum ℓ, are arranged in *configurations* according to shells (n) and subshells (nl), denoted as 1s, 2s, 2p, 3s, 3p, 3d . . . (the number of electrons in each subshell is designated as the exponent). The chemical properties of elements are well-known. Noble gases, such as helium, neon and argon, have low chemical reactivity owing to the tightly bound closed shell electronic structure: $1s^2$ (He, $Z = 2$) $1s^22s^22p^6$ (Ne, $Z = 10$) and $1s^22s^22p^63s^23p^6$ (argon, $Z = 18$). The alkalis, lithium (Li, $Z = 3$), sodium (Na, $Z = 11$), potassium (K, $Z = 19$), etc., have relatively high chemical reactivity owing to the single valence electron ns outside a closed shell configuration, e.g., $1s^22s^1$ (Li) (see Chapter 2 for a detailed discussion). Chemical reactivity is responsible for molecular formation and processes. Sodium or potassium atoms combine easily with chlorine, a halogen with a vacancy in the 3p electronic orbit ($1s^22s^22p^63s^23p^5$), to form NaCl or KCl (common salts); the pairing is through an ionic bond, reflecting the fact that the Na atom 'donates' an electron, while the chlorine atom gains an electron to fill the 'vacancy' to close the outer shell. The chemical properties involving valence electrons and the reactivity of an element are determined by the *electron affinity*, the energy required to remove valence electrons. Atoms with more than one valence electron in an open shell form molecular bonds in a similar manner. The carbon atom has two valence electrons in the 2p shell, which can accommodate six electrons as a closed shell. The four vacancies can be filled by single electrons from four H atoms to form one of the most common molecular compounds in nature, CH_4 (methane), which, for instance, is probably the predominant constituent of 'oceans' on Saturn's moon Titan. Carbon monoxide, CO, is one of the most abundant molecular species in astronomical sources. Its stability lies in the match between the two valence electrons in the carbon atom and the two vacancies in the oxygen atom, which has four electrons in the 2p shell. In general, the chemical properties of elements are concerned with valence electrons and shells of atoms.

On the other hand, by physical properties of elements, we refer largely to spectroscopic and atomic processes, such as energy level structure, radiative transitions, excitations, ionization and more. Of course, these are also based on the electronic structure of atoms and ions but in a different manner than those of chemical processes. To begin with, the physical and chemical properties are expected to be similar for elements along the *columns* of the periodic table, since the electronic structures are similar (discussed in detail in Chapter 2). For example, boron (B) and aluminium (Al) both have a single valence electron in the p shell, preceded by an inner two-electron filled s shell: $1s^22s^22p^1$ (B) and $1s^22s^22p^63s^23p^1$ (Al). Therefore, the energy-level structure and processes involving those levels are usually similar. Both boron and aluminium display two-level fine structure splitting of the ground state $np\left({}^2P^o_{1/2} - {}^2P^o_{3/2}\right)$. Transitions between these two levels generate a weak 'forbidden' spectroscopic line in both the elements. Likewise, the atoms of flourine and chlorine in the halogen column have energy-level structures and spectral features similar to B and Al, owing to the fact that a single-vacancy p shell has the same spectral composition as a single-valence p electron: $1s^22s^22p^5$ (F) and $1s^22s^22p^63s^23p^5$ (Cl); both atoms also have the same ground state as B and Al, ${}^2P^o$, and the same type of forbidden transition.

From the point of view of atomic and astrophysical spectroscopy one of the most important manifestations of physical properties is for ions along an *isoelectronic sequence*: ions of different elements and atomic number Z, but with the same number N of electrons. For example, the helium isolectronic sequence consists of the ions of all elements of the periodic table stripped down to two electrons: $1s^2$ in the ground state (*He-like ions*. The columns of the periodic table already provide a guide to similarity of physical properties, for if similar electronic structure leads to similar properties, then the same electronic structure should do so also. For example, the singly charged carbon ion (expressed by C^+ or C II) has five electrons, isoelectronic with boron; similarly the nitrogen ion N III, the oxygen ion O IV and each ion of an element ($Z > 5$) with five electrons belongs to the boron sequence. However, there is a crucial physical difference with neutral elements along a column in the periodic table: not only is the atomic number Z different, but also the charge on each ion $+z = Z - N$ in the isoelectronic sequence is different. Therefore, the atomic physics, which depends basically on the electromagnetic potential in the atom or ion, is different for each ion. As Z increases, the attractive electron–nucleus Coulomb potential increases, resulting in higher-speed electrons. When the velocities are sufficiently high, relativistic effects become important. The energy-level splittings and processes dependent on relativistic and inter-electron interactions lead to significant differences in spectral formation for ions within the same isoelectronic sequence. We shall discuss a number of aspects of isoelectronic sequences in much more detail in later chapters.

Physical properties of elements also refer to interaction of radiation with matter on the atomic scale, which brings forth some physical processes not usually within the realm of chemistry, such as excitation and ionization of electrons.[1] Finally, physical phenomena are dependent

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[1] To some extent the distinction between physical and chemical processes, as we have drawn here, is superficial from a fundamental viewpoint. But we do so purposefully to emphasize the physical nature of elemental species as they lead to atomic and astrophysical spectroscopy.

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on environmental properties, primarily the temperature and density of the ambient plasma medium. The diversity of astrophysical environments makes it necessary to consider the intrinsic physical properties of atoms in conjunction with extrinsic plasma parameters. This book is concerned with the physical properties of elements in various ionization stages, particularly from an astrophysical perspective.

1.3 Electromagnetic spectrum and observatories

Astrophysical observations using state-of-the-art spectrometers on-board space missions and on ground-based telescopes are revealing spectral features at very high resolution in all wavelength ranges. Indeed, one may view astronomical sources as 'astrophysical laboratories' for atomic physics – a reversal of roles that greatly enhances the reach of both disciplines, atomic physics and astronomy.

Radiation emission from astronomical objects ranges over the whole electromagnetic spectrum from radio waves to gamma rays. The photon energy $h\nu$ and wavelength λ corresponding to each type of radiation are related inversely as

$$\nu = c/\lambda. \tag{1.1}$$

The least energetic radio wave photons have the longest wavelength $\lambda > 1$ m, and the most energetic gamma rays have wavelengths more than ten orders of magnitude smaller, $\lambda < 0.1$ Å. (Note that $1\,\text{Å} = 10^{-10}\,\text{m} = 10^{-4}\,\mu\text{m} = 0.1\,\text{nm}$, where a μm is also referred to as *micron* and 'nm' refers to nano-metre.)

Figure 1.2 is a schematic representation of the different regions of the electromagnetic spectrum of solar radiation transmitting through the terrestrial atmosphere. The atmosphere blocks out most regions of the spectrum (shaded area), except the optical or visible (vis), the near infrared (NIR), and the radio waves. The visible band is, in fact, a very narrow range in wavelength, but of course the one most accessible. The shaded regions are opaque to an observer on the ground, owing to higher atmospheric opacity. For example, water vapour in the atmosphere is very effective in blocking out IR radiation, owing to absorption by H_2O molecules, except in a few 'windows' or bands around 100–1000 nm or 1–10 μm (discussed later). This atmospheric 'blanketing' is also beneficial to us since it not only retains the re-radiated energy from the Earth (the greenhouse effect), but also absorbs the more energetic radiation from the Sun. Even a little more of the Sun's ultraviolet (UV) radiation could be biologically disastrous, not to mention the effect of high energy particles and other cosmic radiation of shorter wavelengths, which, although some do get through, are largely blocked out by the atmosphere. The use of ground-based telescopes is, therefore, confined to the wavelength ranges accessible from the Earth, after propagation of radiation through the atmosphere. For all other wavelengths we need to go into Outer Space.

Figure 1.2 also shows the general division of the electromagnetic spectrum for the Earth-based and space-based telescopes. Satellite-based space observatories make observations in the opaque regions. Some recent space observatories are the Compton Gamma-Ray Observatory (GRO), the X-ray Multi-Mirror Mission-Newton (XMM-Newton), the Chandra X-ray Observatory (CXO), Hubble Space Telescope (HST), Spitzer Infra-red Observatory, etc., respectively named after famous scientists: Arthur Compton, Isaac Newton, Subrahmanyan Chandrasekhar, Edwin Hubble and Lyman Spitzer. Another current mission includes the multi-wavelength X-ray–γ-ray NASA satellite Swift, to study gamma-ray bursts that are found to occur all across the sky, and X-ray observations from active galactic nuclei and other sources.

There is significant overlap in the approximate wavelength ranges given, depending on the detectors and instrumentation. Ground-based telescopes have sensitive spectrometers that can range somewhat outside the range

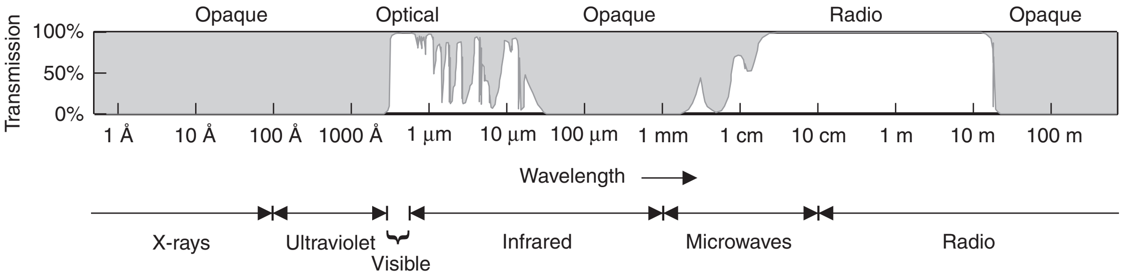

FIGURE 1.2 The electromagnetic spectrum of transmitted radiation through the Earth's atmosphere (http://imagine.gsfc.nasa.gov).

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visible to the human eye, 4000–7000 Å. Optical CCDs (charge-coupled devices) can detect radiation from near UV to near IR, 3000–10 000 Å. The detector capability, measured in terms of the percentage of photons detected, called the *quantum efficiency*, deteriorates rapidly near the edges of some spectral windows. Subsequent chapters on astronomical objects will describe the prominent atomic species and spectral features. Atomic and molecular processes play the dominant role at all wavelengths except gamma-rays due to nuclear processes and electron–positron annihilation or synchrotron radiation.

Exercise 1.1 *Compile a list of current major ground and space observatories with spectroscopic instruments and corresponding wavelength ranges.*

1.4 Astrophysical and laboratory plasmas

Ionized materials in astrophysical plasmas constitute over 99% of the observed matter in the Universe – that is, all the matter in stars, nebulae and interstellar matter, which comprise observable galaxies.[2] As we mentioned, the analysis of characteristic light is the science of spectroscopy, and nearly all information on observable matter is derived from spectroscopy. This is how we really see the Universe in all its glory. Observable matter spans a huge range in density–temperature parameter space. Whereas the interstellar medium may be cold and thin, down to a few K and to less than one particle per cm^3, highly energetic plasmas in the vicinity of black holes at centres of galaxies may approach a thousand million K and immense (as yet unknown) densities. An important set of temperature–density combinations is the one in stellar cores: exceeding ten million K and 100 g cm^{-3}, conditions required for hydrogen nuclear fusion that provides most of stellar energy.

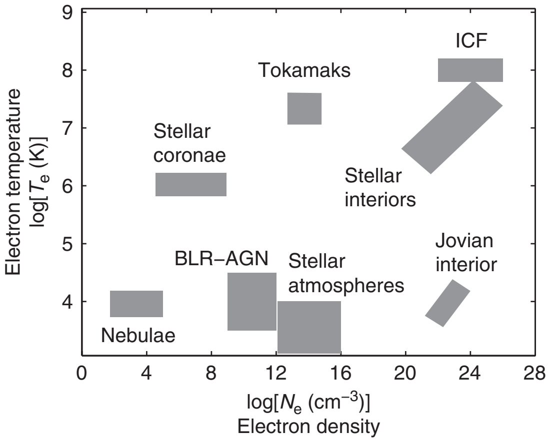

FIGURE 1.3 Temperature–density regimes of plasmas in astrophysical objects, compared with laboratory plasmas in magnetic confinement fusion devices, such as tokamaks, and inertial confinement fusion (ICF) devices, such as Z-pinch machines and high-powered laser facilities. BLR-AGN refers to 'broad-line regions in active galactic nuclei', where many spectral features associated with the central massive black hole activity manifest themselves.

Figure 1.3 shows astrophysical and laboratory plasma sources and their approximate temperatures and densities. As one might see, the astrophysical objects correspond to several regimes of electron temperature T_e and density n_e. Often, only some parts of a source are observed. Ordinary stars, for instance, range from a temperature $T \sim 2000$–3000 K in their outer atmospheres to $>10^7$ K in the core, where thermonuclear fusion creates their energy. The directly observable parts of a star are its *photosphere*, from which most of its radiation is emitted, and the hot highly ionized gas in the *corona*, a tenuous but extended region surrounding the main body of the star. In an extreme manifestation of temperature ranges, a stellar condition called supernova begins with an explosive plasma ball of some thousand million degrees, to less than 10^3 K after a few years of expansion into a 'nebular' remnant of the diffuse ionized plasma. It contains mainly H II and the material ejected from the progenitor star, as well as matter swept up from the interstellar medium. The detailed temperature–density–abundance–ionization structure of objects is revealed by spectral analysis of the observable regions of each type of object in different wavelength ranges, as discussed in individual chapters.

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[2] It is worth mentioning how astronomers currently view matter and energy. There is considerable evidence that observable matter comprises only 4% of the Universe. About 22% is so-called 'dark matter' that apparently does not interact with electromagnetic radiation to emit or absorb light, and is therefore not observed. The existence of dark matter may be inferred by its gravitational influence on objects. For example, the rotation rate of matter within galaxies is observed *not* to decrease with increasing distance from the centre as expected, but rather remains roughly constant to very large distances. This implies that there is unseen matter in and beyond the observable halo of galaxies. Some of the matter may also be hidden in hot and highly ionized gas in the intergalactic medium, which is indicated by X-ray spectroscopy. The remaining 74% constituent is called *dark energy*, if one interprets the observed acceleration in the expansion rate of the Universe as part of the gravitational mass-energy balance. We discuss these topics in detail in Chapter 14.

1.4.1 Laboratory astrophysics

Is it possible to create conditions in the laboratory that simulate those in astrophysical sources? If yes, then it would be possible to study precisely the physical processes at play as well as measure the basic physical quantities that would enable modelling or numerical simulation of the source plasma in general. As we have seen, owing to the vast range of conditions prevailing in astrophysical sources, that is only possible under the restricted conditions available in a laboratory. In fact, experimental conditions are often quite stringent. The temperatures, densities and particle energies must be precisely measurable to obtain meaningful results for physical parameters of interest, such as cross sections and rates. Nevertheless, such experiments constitute what is called *laboratory astrophysics*. Laboratory devices need to create and maintain a plasma for sufficiently long periods of time for measurements to be carried out. This is quite difficult, especially for measuring *absolute* cross sections with high accuracy, since the results need to be calibrated relative to some independent criterion.

Essentially, the experimental techniques are designed to enable electrons and photons to interact with atoms or ions. This is accomplished either by *colliding* or *merging* beams of interacting particles, or in devices that confine an (electron + ion) plasma. Among these instruments for high-resolution measurements are *electron beam ion traps* (EBIT), which create a trapped plasma of ions interacting with an electron beam of controlled energy within some beam width, *ion storage rings*, where ions are magnetically trapped for long periods of time in a ring-like structure, enabling electron-ion experiments, and *advanced light sources* (ALS) of photons mounted on synchrotron accelerators for targeting the 'stored' ions and measuring photoionization cross sections. We will describe these experiments later, while discussing benchmarking and validation of theoretical results.

Laboratory plasma sources, while quite different from astrophysical sources in spatial and temporal behaviour, also span a wide range of temperature–density regimes. Most of the spectral diagnostics and atomic data we describe in this text also apply to laboratory plasmas. In particular, two classes of device for controlled thermonuclear fusion are shown in Fig. 1.3: (i) magnetically confined plasma reactors, called *tokamaks*, and (ii) *inertial confinement fusion* (ICF) devices. The ICF machines are essentially of two types. The first kind is based on laser-induced fusion, wherein an arrangement of symmetrically directed and powerful lasers is fired at a small deuterium-tritium pellet (heavy isotopes of hydrogen containing either one or two neutrons respectively), causing it to implode. The second kind are the so-called Z-pinch[3] machines, wherein a very high electrical discharge passes through wires of a heavy element, arranged cylindrically, which explode and emit X-rays that are directed towards the pellet. The fusion pellet is placed inside the cylindrical formation or cavity, which is called the *hohlraum*. At high temperatures, heavy elements exist in many ionization stages and emit copious amount of radiation; gold ($Z = 79$) is the common choice for hohlraum wires.

1.4.2 Astrophysical plasma composition and abundances

Astronomical objects are generally electrically neutral, i.e., an equal number of negative (electrons) and positive (protons and other ions) charges exists. The electrons are the dominant and most 'active' constituents, since their velocities compared with those of protons are $v_e/v_p = \sqrt{m_p/m_e} = 42.85$. In astronomical plasmas, typical proton densities are $\sim 80\%$ of n_e, and other heavier ions, such as helium nuclei (α particles), and partially or fully stripped ions of heavier elements constitute the rest of the positively charged particles. In astrophysical nomenclature, all elements heavier than helium are called 'metals'. Cosmic plasma compositions denote the H-abundance as 'X', He-abundance as 'Y', and all other metals combined as 'Z'. For instance, the solar elemental composition by mass is X = 0.70, Y = 0.28, Z = 0.02. Although metals constitute only 2% of the plasma, they are responsible for most of the spectral features, and they crucially determine properties, such as the plasma opacity that governs the transfer of radiation through the source (Chapter 10). Further study of plasmas in various situations requires us to consider the fundamental bulk properties associated with this most prevalent state of matter.

1.5 Particle distributions

A plasma of charged particles and a radiation field of photons can be treated with certain distribution functions.

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[3] The 'Z' here refers *not* to the atomic number but the fact that a current passing along the z-axis through a wire creates a surrounding magnetic field, which acts naturally to constrain or 'pinch' the exploding plasma; the wires are indeed made of high-Z material!

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1.5.1 Fermions and bosons

Concepts of indistinguishability and symmetry play a fundamental role in quantum mechanics. All particles of a given kind; electrons, protons, photons, etc., have the same observational property of being indistinguishable from other particles of the same kind. This universal fact is of profound importance and is known as the *principle of indistinguishability* [3]. Quantum mechanically, all observable quantities are expressed in terms of probabilities derived from a wavefunction formed for each kind of particle in terms of its spatial and spin coordinates. But the probabilities are related to the *squares* of the wavefunctions. That introduces an ambiguity in the actual sign of the wavefunction, which can be '+' or '−'. The total wavefunction of an ensemble of identical particles is therefore fixed by nature into two kinds. The first kind, *bosons*, refers to a *symmetric* total wavefunction, corresponding to the fact that interchange of coordinates of any two particles leaves the sign of the wavefunction unchanged. The second kind are called *fermions*, which correspond to an *antisymmetric* total wavefunction that changes sign upon interchange of coordinates.[4] The spin is a special 'coordinate', and has a value that is either integral (or zero), or half-integral. Bosons are particles of zero or integral spin, and fermions possess half-integral intrinsic angular momentum. Bosons and fermions obey different statistical mechanics: *Bose–Einstein* statistics in the case of bosons, and *Fermi–Dirac statistics* for fermions, both discussed in the next section.

1.5.2 Temperature: Maxwellian and Planck functions

The concept of 'temperature', which gives a measure of hot and cold in general sense, needs a more precise description in astronomy. For a given system of particles, say photons or electrons, the temperature has a meaning if and only if it corresponds to a distinct radiation (photon) or particle (electron) energy *distribution*. In the ordinary sense, the 'temperature' of a photon or an electron, or even a photon or electron beam, is meaningless. But the root-mean-square (rms) particle energy may be simply related to the *kinetic* temperature according to

$$E = h\nu \sim kT, \text{ and } E = 1/2\, mv^2. \tag{1.2}$$

With three-dimensional compoments of the velocity,

$$\frac{1}{2}mv^2 = \frac{3}{2}kT, \; k = 8.6171 \times 10^{-5}\ \text{eV K}^{-1} \\ = 1.380\,62 \times 10^{-16}\ \text{erg K}^{-1} \tag{1.3}$$

where k is the Boltzmann constant. Consider a star ionizing a molecular cloud into a gaseous nebula. *Nebulae* are a class of so-called H II regions where the principal ionic species is ionized hydrogen (protons). The two distinct objects, the star and the nebula, have different temperatures; one refers to the energy of the radiation emitted by the star, and the other to the energy of electrons in the surrounding ionized gas heated by the star.

1.5.2.1 *Black-body radiation and the Sun*

The total energy emitted by an object per unit area per unit time is related to its temperature by the *Stefan–Boltzmann Law*

$$E = \sigma T^4, \quad \text{where } \sigma = 5.67 \times 10^{-8}\ \text{W}\,(\text{m}^{-2}\ \text{K}^{-4}) \tag{1.4}$$

is known as the Stefan constant. The Stefan–Boltzmann relation holds for a body in *thermal equilibrium*. The term *black body* expresses black colour or rather the lack of any preferred colour, absorbing radiation most efficiently at *all* wavelengths. *Kirchhoff's law* states that the emissivity of a black body is related to its absorptivity; a black body is also the most efficient radiator (emitter) at all wavelengths (discussed further in Chapter 10). At any temperature, a black body emits energy in the form of electromagnetic radiation at *all* wavelengths or frequencies. However, the distribution of emitted radiation changes with temperature such that the peak value and form of the distribution function defines a unique temperature for the object (black body), as discussed in the next section. The total *luminosity* L of a spherical black body of radius R, such as a star, integrated over all frequencies, is called the *bolometric luminosity*,

$$L = 4\pi R^2 \sigma T^4. \tag{1.5}$$

The radiation field of a star, considered to be a black body, is given by the Planck distribution function,[5] which defines the energy–frequency relationship at a given temperature:

$$B_\nu(T_*) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT_*) - 1}, \tag{1.6}$$

where T_* is the *radiation temperature* of the star and ν is the frequency of the photons. In terms of wavelengths it reads

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[4] A simple and elegant 'proof' is given in the classic textbook by E. U. Condon and G. H. Shortley [3].

[5] This is the underlying radiation field, which is attenuated by spectral features, such as lines and bands particular to the star.

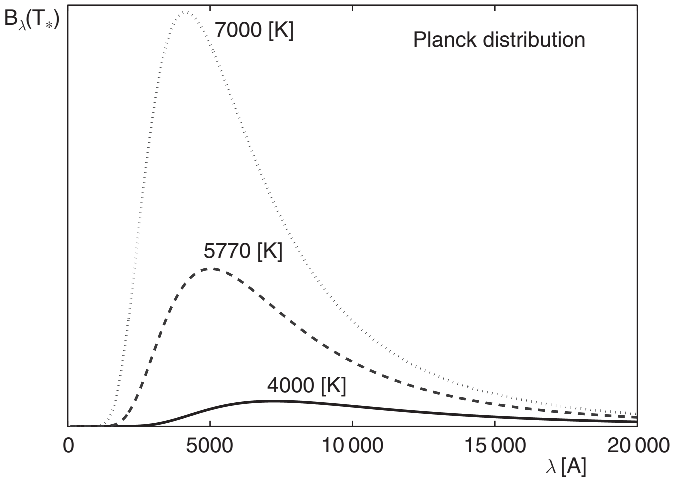

FIGURE 1.4 Planck distributions of photon intensity vs. wavelength at radiation temperatures T_* of various stars. Light from the Sun corresponds to $T_* = 5770$ K, which peaks at wavelengths around yellow; stars with higher(lower) temperature are bluer (redder). The Planck function B_λ is discussed in the text.

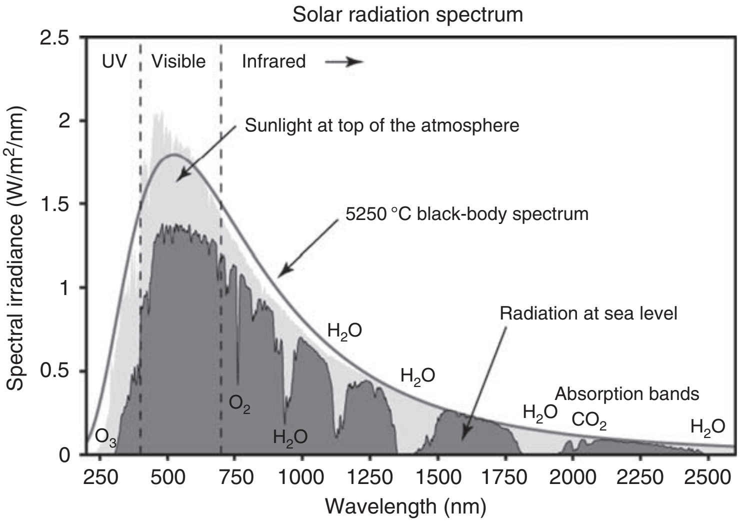

FIGURE 1.5 Sunlight as received at the top of the atmosphere and at sea level after attenuation by atmospheric constituents, primarily ozone, oxygen, water and carbon dioxide. (Courtesy Robert A. Rhode, http://globalwarmingart.com/wiki/Image:Solar_Spectrum.png).

$$B_\lambda(T_*) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda kT_*) - 1}. \tag{1.7}$$

A surface temperature of $T_* = 5770$ K corresponds to the peak emission of a black body in the characteristic colour of the Sun – yellow – around 5500 Å. Hotter stars radiate more in the blue or ultraviolet and are 'bluer'; cooler stars radiate a greater fraction in the infrared and are 'redder' than the Sun. Figure 1.4 shows the black-body curves for several temperatures T_* representative of stars. *Wien's law* states that the black body distribution $B_\lambda(T)$ peaks at

$$\lambda_{\rm p} = \frac{2.8978 \times 10^7\ \text{Å}}{T/\text{K}}, \tag{1.8}$$

thus peaking at $\dfrac{2.898 \times 10^3\ \text{Å}}{T/10\,000\ \text{K}}$ or 2900 Å at 10 000 K.

We have already noted the historical relevance of the spectrum of the Sun. But, of course, the Sun is of great importance otherwise. It is therefore instructive to introduce a few salient features of solar spectra, some of which we shall deal with in later chapters.

Figure 1.5 illustrates several aspects of 'sunlight' as received on the Earth. First, following the discussion above about black-body curves associated with the radiation of stars, Fig. 1.5 is fitted to a black body at a slightly lower temperature, 5250 K, than at the surface of the Sun. The best fitting Planck function corresponds to a somewhat lower temperature than the actual spectrum observed *above* the atmosphere (light grey). The spectrum at *sea level* (dark grey) is seen to be significantly attenuated by absorption by the constituents of the atmosphere, primarily molecular bands due to water, oxygen and ozone. Figure 1.5 also shows that, although the peak of Sun's radiation is in the visible, there is a long tail indicating significant flux in the infrared. Water vapour in the atmosphere absorbs much of the longer wavelength ($\lambda > 1000\,\text{nm} = 10\,000\,\text{Å} = 1\,\mu\text{m}$) IR radiation via molecular transitions in H_2O.[6] But water also allows a considerable amount of solar radiation to be transmitted through the atmosphere through what are referred to as 'windows' in certain wavelength bands where H_2O has inefficient absorption (weak molecular transitions). Three of these windows are of particular importance, since they enable astronomical observations to be made from ground level in these IR bands, referred to as the *J, H and K bands*[7] centred around 1.2 μm, 1.6 μm and 2.2 μm, respectively.

Another interesting feature of Fig. 1.5 is the difference in radiation above and below the atmosphere on the UV side where, unlike the IR, the solar UV flux drops off rapidly. This is the *ozone effect*, as O_3 prevents the harmful UV radiation from reaching the Earth and thereby makes life as we know it possible.

1.5.2.2 *Maxwellian particle distribution*

Using old quantum theory (before the invention of wave mechanics), Einstein proposed an explanation of the *photoelectric effect* that relates Planck's quantum of energy $h\nu$ to absorption by an atom with the ejection of an electron. For instance, if the atom is surrounded by other atoms as in a metal, then a certain amount of energy is needed for the electron to escape. Hence the kinetic energy of the photoelectron is obtained as

$$\frac{1}{2}mv^2 = h\nu - W, \tag{1.9}$$

where W is called a *work function*. In the process of photoionization, where an atom or ion is ionized by absorbing a photon, W may be thought of as the *ionization energy* E_I of a bound electron.

The charged particles in the plasma ionized by a star in an H II region have an *electron temperature* T_e associated with the mean kinetic energy of the electrons given by Eq. 1.2. But it makes little sense to refer to the temperature of a single particle. Hence an averaged kinetic energy over a specified distribution of particle velocities is defined. In most astrophysical sources the fractional probability of electrons as a function of velocity or energy is characterized by a Maxwellian distribution of electrons at temperature T_e as

$$f(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right). \tag{1.10}$$

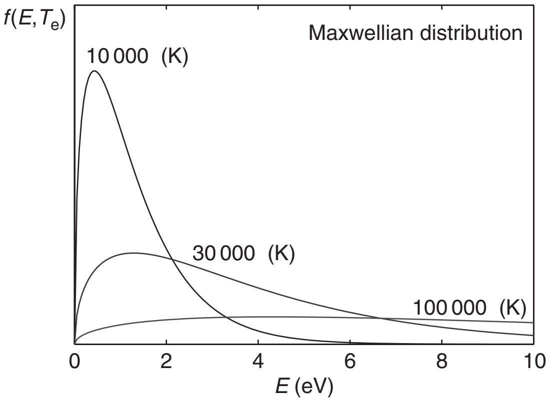

FIGURE 1.6 Maxwellian distributions, $f(E,T_e)$ of free electron energies at three bulk plasma kinetic temperatures T_e.

Figure 1.6 shows the general form of the Maxwellian distribution functions at a few characteristic temperatures T_e. An example of the distinction, as well as the physical connection, between the Planck function and the Maxwellian function is found in H II regions. They are ionized by the hottest stars with black-body temperatures of $T_* \approx 30\,000$–$40\,000$ K. The resulting ionization of hydrogen in a molecular cloud (see Chapter 12) creates a plasma with electron kinetic energies that can be described by a Maxwellian distribution at $T_e \approx 10\,000$–$20\,000$ K. Since $1\,\text{eV} \equiv 11\,600\,\text{K}$ (see Eq. 1.3), the electron temperature is of the order of 1 eV in H II regions.

Plasmas need not always have a Maxwellian distribution; electron velocity (energy) distributions may not be given by Eq. 1.10. For example, in the expanding ejecta of supernovae, solar flares or laboratory fusion devices, some electrons may be accelerated to very high velocities. Such non-Maxwellian components or high-energy 'tails' may co-exist in a source otherwise characterized by a Maxwellian plasma that defines the bulk kinetic temperature. Another example is that of mono-energetic beams used in laboratory experiments; the beam widths may be described by the well-known Gaussian distribution centred around a given energy. It should be noted that often the subscripts on temperature T are omitted, and it is the context that determines whether the reference is to radiation or the electron temperature, T_* or T_e. The kinetic

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[6] The basic composition of the Earth's atmospheric gases by volume at sea level is $N_2 \sim 78\%$, $O_2 \sim 21\%$. Note that these two gases alone comprise ~99% of the dry atmosphere. But there are variations, allowing for $H_2O < 4\%$ and $CO_2 \sim 0.036\%$, and some other trace gases. Both H_2O and CO_2 are greenhouse gases that regulate the greenhouse effect on Earth. Although the CO_2 concentration is usually only about one hundredth of that of H_2O, it can be pivotal in global warming, since it is directly affected by life on Earth and carbon-based fossil fuels.

[7] Not to be confused with the H and K lines of ionized calcium.

temperature of other particle constituents in a plasma, such as protons or other ions ($T_{\mathrm{p}},\ T_{\mathrm{i}}$), is also characterized in terms of a Maxwellian. However, it may happen that $T_{\mathrm{e}} \neq T_{\mathrm{p}}$ or T_{i}, if there are bulk motions or processes that separate electrons from protons or ions.

1.6 Quantum statistics

Free particles, such as fermions and bosons, usually obey the Maxwellian or Planckian distributions associated with a temperature. When particles congregate to form structures like atoms, molecules, etc., they do so in accordance with laws of quantum mechanics described by energy levels quantized in energy and other variables such as momentum. The statistical mechanics of quantum distribution of particles among those levels is *quantum statistics*. There are three statistical distributions that relate to plasma sources. Once again, temperature is the crucial variable that determines the energies of particles and the levels they can occupy, subject to the principle of indistinguishability (and hence their fundamental classification as fermions or bosons) and quantum mechanical rules, such as the *Pauli exclusion principle*.

1.6.1 Maxwell–Boltzmann statistics

In thermal equilibrium, temperature determines the energy available for particles to be excited to higher levels, and the population distribution among them. Assuming a temperature T and a given excited energy level E_i, the distribution of the number of particles in level i relative to the total is

$$\frac{N_i}{N} = \frac{g_i\,\mathrm{e}^{-E_i/kT}}{\sum_j g_j\,\mathrm{e}^{-E_j/kT}}. \tag{1.11}$$

Here, g_i is the statistical weight for level i or its maximum possible occupancy number. The Maxwell–Boltzmann distribution is the one most frequently used to evaluate the number of electrons in excited levels of an atom or ion. The denominator in Eq. 1.11 is referred to as the *partition function*,

$$U = \sum_i g_i\,\mathrm{e}^{-E_i/kT}. \tag{1.12}$$

It is related to what is known as the *equation-of-state* for a plasma and is discussed in detail in Chapter 10.

1.6.2 Fermi–Dirac statistics

What happens as $T \to 0$? In that limit, the distribution of particles depends on their basic nature; fermions or bosons. Since fermions are particles with half-integral spin they must occupy discrete states in accordance with the Pauli exclusion principle, which states that no two fermions can occupy the same quantum mechanical state. This basic fact leads to atomic structure, corresponding to the states of the atom defined by the couplings of angular and spin momenta of all electrons (Chapter 2). As the temperature approaches absolute zero, the electrons have no energy to be excited into higher levels. But not all atomic electrons can occupy the same quantum mechanical state, in particular the ground state, since that would violate the exclusion principle. So they occupy the next available higher levels, until a highest level, called the *Fermi level*, with energy E_{F}. The Fermi–Dirac probability distribution is given by

$$f(E_i, T) = \frac{1}{\exp[(E_i - E_{\mathrm{F}})/kT] + 1}. \tag{1.13}$$

At $T = 0$, we have probabilities $f(E, T) = 1$ if $E \leq E_{\mathrm{F}}$ and $f(E, T) = 0$ otherwise. We may visualize the situation as in Fig. 1.7. All levels up to the Fermi level are filled at absolute temperature zero, constituting an ensemble of fermions called the *Fermi sea*. As T increases, particles get excited to higher levels, out of the Fermi sea. Eventually, for sufficiently high temperature and $kT \gg E$, the Fermi–Dirac distribution approaches the Maxwell–Boltzmann distribution characterized by the exponentially decaying probability as $\exp(-E/kT)$ in Eq. 1.11. The probability $\big($Eq. 1.13$\big)$ is related to the actual number of particles in an energy level i as

$$N_i(\mathrm{FD}) = \frac{g_i}{\exp[(E_i - E_{\mathrm{F}})/kT] + 1}. \tag{1.14}$$

Thus far we have considered only the temperature as the primary physical quantity. But in fact the density of the plasma plays an equally important role. Intuitively one can see that for sufficiently high densities, at any temperature, particles may be forced together so that the exclusion principle applies. In such a situation one can think of a ‘quantum degeneracy pressure’ owing to the fact that no two electrons with all the same quantum numbers can be forced into the same state. When that happens, all accessible levels would again be occupied at the given temperature *and* density. The foremost example of Fermi–Dirac distribution in astrophysics is that of *white dwarfs* Chapter 10). These are stellar remnants of ordinary stars, like the Sun, but at the end of stellar evolution after the nuclear fuel (fusion of H, He, etc.) that powers the star runs out. The white dwarfs have extremely high densities, about a million times that of the Sun. The electrons in white dwarfs experience degeneracy pressure, which in fact prevents their gravitational collapse by forcing the