# Frontiers in Applied General Equilibrium Modeling

*In Honor of Herbert Scarf*

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# Introduction

Timothy J. Kehoe, T. N. Srinivasan, and John Whalley

This volume honors Herbert Scarf and his contributions to economics. It deals with new developments in applied general equilibrium (AGE) modeling, a field in which Scarf's contributions have played a decisive role. All but two of the chapters in the volume were presented at a conference held at Yale University in April 2002. The chapter by Herbert Scarf and Charles Wilson was written afterward; it demonstrates the uniqueness of equilibrium in an important class of international trade models. The chapter by Lars Ljungqvist and Thomas Sargent is an outgrowth of Sargent's discussion at the conference of the paper presented there by Edward Prescott. The chapters presented here build on a well-known earlier volume in applied general equilibrium, edited by Herbert Scarf and John Shoven in 1984 (Scarf and Shoven 1984), which in turn grew out of Scarf's pioneering contributions in general equilibrium computation in the 1960s and early 1970s (Scarf 1967a, Scarf and Hansen 1973). Kenneth Arrow's chapter in this volume points out that the ability to deploy AGE models is the product of research advances, going back at least 130 years, in which progress in economic theory and vastly improved availability of economic data have played crucial roles. According to Arrow, equally crucial inputs were improvements in computing power and the development of algorithms for computing equilibrium, in which Scarf's (1967b) algorithm based on simplicial subdivisions was the crucial step.[1]

Since the 1980s, applications of AGE have broadened. They now include international trade, public finance, development, energy, and climate change and broader environmental concerns, as well as other fields. A range of new approaches and conceptual issues, not to mention computational algorithms, has evolved. These include calibration and expanded areas of application, such as macroeconomics of real business cycles and finance. In addition, the techniques of AGE modeling – namely calibrating and benchmarking observed data on economies into an initial

[1] In an as yet unpublished paper, Scarf (2002) provides a fascinating account of his involvement in the computation of economic equilibria and the contribution of his interaction with faculty and students at Yale in the late sixties and seventies.

equilibrium data set and then doing counterfactual policy analysis – have spread into other areas, such as game theory and even partial equilibrium models of industrial organization.

After the initial phase of demonstrating the potential of using AGE models for policy analysis, searching questions were raised as to the performance and robustness of AGE as a basic tool in policy and other work. Most of the policy applications had been performed ex ante, in anticipation of a policy change being enacted, such as the implementation of NAFTA or of the Uruguay Round agreement on international trade. These applications provided valuable estimates of the likely consequences of a policy change. For the methodology of AGE to become a widely accepted and useful policy tool, however, cross checking of the projections of models with actual outcomes, after the policy change has been put in place, is essential. Such cross checking (which is an analogue of cross checking of out-of-sample predictions of an econometric model with its actual realizations) has to allow for the fact that projections of an AGE model are conditional in that they are based on particular assumptions about values of variables exogenous to the model, and, as such, the projections could deviate from the actual outcomes if the realized values of exogenous variables differed from their assumed values. Also, in actual implementation, aspects of a policy could differ from those assumed in the model, and other policies not included in the model could be implemented at the same time. Nonetheless, with appropriate allowance for these factors, it should be possible to look backward, after the model's policy change has been implemented, and evaluate how accurate and useful the model projections were. Timothy Kehoe's evaluation of models of NAFTA in this volume is one such attempt.

This volume builds on existing AGE literature and consciously aims to go well beyond it and to look to the future. Scarf's research agenda of making the elegant theoretical general equilibrium models fully operational, implementable with actual data, and useful to practitioners such as policy makers is relevant to all theoretical models of economics. All analytical structures should, in principle, have their numerical analogues implementable with data. The practical issues are how to do this and what conclusions can be drawn from simulations or projections from the numerical model. Similar issues of how models are parameterized or calibrated arise even when models other than the general equilibrium model are used.

The chapters in this volume illustrate both the progress in AGE modeling since the 1980s and applications to new areas, as well as challenges that remain to be addressed. We start with a discussion of the origins of applied general equilibrium modeling and Herbert Scarf's contributions to this field. We then provide brief descriptions of the individual chapters.

## ORIGINS OF APPLIED GENERAL EQUILIBRIUM MODELING

Numerical applications of general equilibrium began with the work of Arnold Harberger (1962) and Leif Johansen (1960). Harberger used a model with two

production sectors, one corporate and one noncorporate, calibrated to U.S. data from the 1950s, to calculate the incidence of the U.S. corporate income tax. Johansen used a model with nineteen production sectors, calibrated to Norwegian data from 1950, to identify the sources of economic growth in Norway over the period 1948–53. Both linearized the model and solved it analytically without worrying whether an equilibrium of the original nonlinear model actually existed near the benchmark equilibrium. Neither Harberger nor Johansen raised the possibility of multiple equilibria in the model or attempted to check for multiplicity in any way. Interestingly, in many of the more recent contributions to AGE, proofs of existence are also forgone, and instead a computational algorithm is presented, which, in practice, converges to an approximate equilibrium, given some specified measures for closeness of approximation. The check for multiplicity, if at all attempted, is often rudimentary – it is simply whether the algorithm, starting from different initial positions, converges to the same or different final positions.

The first rigorous approach to developing a computational algorithm that was guaranteed to find equilibria to any desired degree of approximation dates to the pioneering work of Herbert Scarf, first published in 1967. Although Scarf himself did not ever put together an AGE model and solve it for its equilibrium using his algorithm, he clearly had applications in mind. In fact, describing his involvement in the computation of equilibrium, Scarf (2002) has this to say about the numerical example with six commodities and eight activities in his 1967 paper (Scarf 1967a):

> The example was meant to suggest to my colleagues, at Yale and elsewhere, that these novel numerical techniques might be useful in assessing consequences for the economy of a change in the economic environment, or in a major policy variable – to engage in comparative statics where the equilibrium model was too large to solve graphically or by hand.

He adds that:

> . . . it was some time before this suggestion was taken seriously. We were in the 1960s, in the era of large Keynesian macro models in which specific scarce resources and relative prices were not included; there was, in the air, a suggestion that the economy could actually be fine-tuned by prescient economic advisors.

It turned out that building large macro econometric models would no longer engage academic macroeconomists, although private economic forecasters and some public agencies continue to use such models. Whether this development is to be applauded or regretted, it is a fact. In contrast, the use of AGE models has grown far beyond what Scarf might have foreseen in 1967.

We would like to supplement the intellectual history of AGE modeling, narrated by Arrow in his paper, with an account of the contributions of Scarf and his students at Yale to this history, drawing on the account in Scarf (2002). Kenneth Arrow and Gerard Debreu (1954) and Lionel McKenzie (1959) provided a careful definition of a competitive equilibrium, a rigorous proof of its existence under certain sufficient conditions, and a characterization of the equilibrium (often called the two

fundamental theorems of neoclassical welfare economics) and its extension to cover transactions over time and involving uncertainty. The first fundamental theorem established that the set of competitive allocations without lump sum redistribution is a subset of Pareto efficient allocations. The second theorem shows that with lump sum redistributions, any Pareto efficient allocation can be sustained as a competitive equilibrium, thus the set of competitive allocations and the set of Pareto efficient allocations are the same, once lump sum redistributions are allowed. In 1881, Francis Edgeworth had developed the idea of the core as the set of allocations upon which no coalition of agents in the economy can improve, in the sense of doing better for all of its members by an alternate allocation in an economy of its own with its own endowments and technology. The core is obviously a subset of the set of Pareto efficient allocations, but in general, the core is much smaller than the Pareto efficient set. Debreu and Scarf (1963) proved a deeper result than the first welfare theorem by showing that the core converges to the set of competitive allocations – without lump sum income redistributions – as the economy is replicated.

The issue of whether there is a mechanism that will lead an economy to a competitive equilibrium is related to the development of an algorithm to compute equilibrium. Léon Walras, the founding father of general equilibrium theory, had proposed in 1874 a process that he called *tâtonnement*, or groping, to find an equilibrium. Paul Samuelson later formalized this *tâtonnement* process as a system of differential equations. This process raises the price of a good in positive excess demand and lowers the price of a good in negative excess demand. Research in the late 1950s by Kenneth Arrow, H. D. Block, and Leonid Hurwicz (1959) and by Hirofumi Uzawa (1960) established that the Walrasian *tâtonnement* process for an exchange economy was globally stable provided either that the market excess demands exhibited gross substitutability or that they satisfied the weak axiom of revealed preference. Unfortunately, although gross substitutability in the excess demand of each individual consumer ensures that it holds also for the aggregate market excess demand, it is not satisfied for individual demand functions that exhibit some complementarity, however modest. On the other hand, although the weak axiom is satisfied by individual excess demands, it need not be satisfied by aggregate excess demands. In 1960, Scarf produced the first examples of global instability of the competitive equilibrium woven around preferences that exhibit complementarity. Scarf's examples come as no surprise now, because a later series of papers by Hugo Sonnenschein (1973), Rolf Mantel (1974) (a student of Scarf), and Gerard Debreu (1974) showed that, with a sufficient number of consumers, aggregate excess demand is essentially arbitrary and hence the behavior of the *tâtonnement* can be made to follow arbitrary curves. At the time, however, Scarf's (1960) paper had considerable influence in discouraging enthusiasm for the *tâtonnement* process. It also had the effect of focusing Scarf's own attention on the need for developing an algorithm for calculating competitive equilibria.

Scarf had first thought that computation of competitive equilibria could be found by finding allocations in the core of an economy and then replicating. He had developed an algorithm for finding allocations in the core (Scarf 1967c) using techniques similar to those developed by Carlton Lemke and Joseph Howson (1964) for finding Nash equilibria for two-person non-zero-sum games. Scarf then developed an alternative algorithm avoiding the core and closely related to Sperner's argument demonstrating Brouwer's fixed point theorem. This later work was the beginning of homotopy, or path-following, computational algorithms for calculating equilibria. Scarf himself later made important contributions to the theory and implantation of these algorithms, most notably in his joint work with Curtis Eaves (Eaves and Scarf 1976).

Some students of Scarf at Yale – Terje Hansen, Timothy Kehoe, Rolf Mantel, Michael Todd, and Ludo van der Heyden – wrote Ph.D. theses on computation or on theoretical topics related to computation. But Scarf encouraged even more of his students to search for ways in which to apply general equilibrium theory and these novel computational techniques. Indeed, there is what many would characterize as the Yale school of economists, who use AGE models to do economic policy analysis. Students of Scarf in this group include Andrew Feltenstein, Timothy Kehoe, Ana Matirena-Mantel, Marcus Miller, Donald Richter, Jaime Serra-Puche, John Shoven, John Spencer, and John Whalley. A feature that characterizes the research of the Yale school of AGE modeling – and distinguishes it from some other AGE modelers – is its heavy interaction with the general equilibrium theory of Arrow, Debreu, McKenzie, and Scarf. Members of the Yale school rely on rigorous theory to guide the development of their models, and they carefully modify and develop new theory when the existing theory is not adequate for their particular applications.

As applications of AGE modeling progressed, they were taken up by governments and international organizations around the world. The World Bank, the World Trade Organization, the International Monetary Fund, and government agencies in the United States, Australia, Canada, Mexico, the United Kingdom, the Netherlands, and many other countries all had general equilibrium models. The field of AGE modeling as an operational tool in government and policy circles was launched.

## CONTRIBUTIONS IN THIS VOLUME

We have grouped the contributions into parts in order to bring coherence to our discussion and to draw on any overlaps among them. Part 1 has two chapters. The first, by Kenneth Arrow, is an expanded version of his talk at the conference dinner. It is a fascinating recapitulation of the intellectual history of general equilibrium theory and its use in AGE. The chapter of Herbert Scarf and Charles Wilson follows Arrow's chapter. It is on pure general equilibrium theory as applied to the classic Ricardian model of international trade. It provides elegant proofs for the uniqueness (assumed by most trade economists) of equilibrium for the model under the well-known

sufficient condition of gross substitution in aggregate demand. One of the proofs relies on the fixed point index theorem developed by Timothy Kehoe (1980) in his Ph.D. thesis, written under Scarf's supervision. Kehoe's work had been inspired by the results obtained by Eaves and Scarf (1976).

Part 2 consists of two chapters on developments in computation methods. The first, by Kenneth Judd, presents an alternative algorithm for solving dynamic stochastic models, combining convergent methods for solving finite systems of equations with convergent dynamic programming. The second, by Michel Ferris, Steven Dirkse, and Alexander Meeraus, describes a new suite of methods for solving problems that combine facets of optimization and complementarity using a unifying framework of mathematical programs with equilibrium constraints.

Part 3 is devoted to applications in macroeconomics and finance. It consists of four chapters. Edward Prescott reviews the role of nonconvexities at the micro level in macro business cycles. In contrast to a long tradition of viewing business fluctuations as disequilibrium phenomena, in contemporary stochastic dynamic general equilibrium macroeconomic models, of which Prescott's is one, the cycles emerge from the stochastic processes that are essential elements of the models. Thus random, but persistent, changes in the factors that determine the level of output give rise to fluctuations that approximate those observed in real economies. The chapter by Lars Ljungqvist and Thomas Sargent both complements and challenges the results presented by Prescott. It shows that models in which unemployment is frictional have very different implications for the data than do models, such as that of Prescott, in which lotteries transform the economic environment into a standard Arrow–Debreu–McKenzie general equilibrium setting. The chapter by Makoto Nakajima and José-Víctor Ríos-Rull focuses on borrowing and lending by individual agents with endogenous default and credit limits and explores the extent to which aggregate events are amplified or smoothed by bankruptcy filings. The parameters of this model are estimated using U.S. data and the model replicates aggregate fluctuation of the U.S. economy. The chapter by Alosio Araujo and Mário Páscoa also models default penalties and collateral and credit restrictions. It extends the received theory of general equilibrium with incomplete markets that has been used to analyze the stochastic volatility of asset prices and the risk premium puzzle to incorporate default, credit risk, and institutions to deal with them. This chapter, which is theoretical, shows that Ponzi schemes and asset price bubbles may occur. The authors provide sufficient conditions for the nonexistence of bubbles in equilibrium.

Part 4 consists of three chapters on applications of AGE to public finance, development, and climate change. Dale Jorgensen and Kun-Young Yun employ an aggregate dynamic general equilibrium model of the U.S. economy to analyze the economic impact of alternative tax reform proposals. Equilibrium is characterized by an intertemporal price system that clears markets for labor, capital services, consumption goods, and investment goods. Starting from the base case solution of a unique steady state for the tax policy existing in 1996 and the associated

transition to that steady state from initial conditions of 1996, they solve for the unique transition path following tax reform to compare social welfare associated with each policy proposal with that in the base case. Jorgenson and Yun find a substantial welfare gain from a reform that they call efficient taxation of income. This reform treats income sources symmetrically, reduces marginal rates, and retains progressivity. The chapter by François Bourguignon, Anne-Sophie Robilliard, and Sherman Robinson provides a methodology for linking a household-based micro simulation model of income generation from labor force participation and occupational choices, given wages and prices, with a sectoral AGE model that determines the commodity and factor prices in equilibrium.[2] The proposed methodology is illustrated with household survey data and sectoral data from Indonesia. The model is used to assess the impacts on income distribution of a terms-of-trade shock that reduces the export price of crude oil and processed oil products and of a shock that reduces external capital inflow by 30 percent. The simulations using the authors' methodology are compared to those from the use of a traditional methodology in which 9,800 sample households are aggregated into ten household types. The comparison suggests that the differences may be quite substantial, in one case even reversing the sign of the impact of the shock on inequality. The chapter by Alan S. Manne uses a ten-region, multiperiod (fifteen decades, starting from the base year of 2000), and multisector model to illustrate the controversial issues in the debate over the United Nations Framework on Climate Change of 1992 and the later Kyoto Protocol of 1997. It provides a perspective on emissions and on taxes to restrain these emissions. The implications of the use of alternative rates for discounting the future and the possible presence (or absence) of equity–efficiency trade-offs are explored.

Part 5 consists of an encyclopedic contribution by James Heckman, Rosa Matzkin, and Lars Nesheim. This chapter tackles the problem of estimation of hedonic models that price differentiated goods or services (such as that of labor) using an equilibrium framework. Because most goods and services traded in an economy are differentiated, understanding the structure of demand and supply of differentiated goods is essential for a normative analysis of policy proposals in such areas as education, occupational safety, and job training as well as a positive analysis of incorporating quality changes into price indices. This task seems daunting because the specification of preferences and technology in models with differentiated goods involves the characteristics of these goods rather than the goods themselves. Although potential applications of hedonic models are myriad, the authors point out that their application and development, except in certain special cases, have been hindered by computational difficulties, failure to exploit the implications of equilibrium in the hedonic model, and the widely held (but erroneous) belief that identification of structural parameters in a hedonic model is not possible using data from a single

[2] The authors are aware of and state explicitly that the methodology involves several ad hoc assumptions. For this and other reasons it is an open question whether the linked model is fully coherent.

market. Heckman, Matzkin, and Nesheim present analytical and computational results for two classes (scalar additive and nonadditive) of hedonic models that fill the gaps in the literature. They simulate and estimate examples of equilibrium and provide evidence on the performance of several estimation techniques. In many ways, the chapter is groundbreaking. It is distinct from other chapters in this volume in its systematic and internally consistent use of the concept of economic equilibrium (loosely speaking, prices clearing markets) and its precise implications for the distribution of the relevant latent variables so that alternative methods of estimation of underlying structural parameters can be conceived of and their performance assessed.

Part 6 is devoted to performance and policy use of AGE models. It consists of three chapters. The chapter by Timothy Kehoe is an evaluation post-NAFTA of the performances of three different multisectoral static AGE models that had been constructed to project ex ante the impact of NAFTA. His findings are sobering – these models drastically underestimated the impact of NAFTA on North American trade and failed to capture much of the relative impact on different sectors. Kehoe concludes that a new theoretical mechanism for generating large increases in trade in product categories with little or no previous trade (as, in fact, happened post-NAFTA) and an approach to capturing changes in productivity are needed for AGE models to project ex ante future outcomes reasonably well.

Since the 1980s, the inequality in the distribution of wages, particularly across workers of varying skills, has increased in the industrializaed countries. Two, not necessarily competing, sources for this trend have been proposed. One is the growth in trade of industrialized countries with labor-abundant less developed countries. The other is skill-biased technical change. Lisandro Abrego and John Whalley evaluate the relative contributions of the two sources to the observed increases in wage inequality in the United Kingdom between 1979 and 1995. They find that the contribution of the second source has been underestimated by other analysts and the contribution of the first is small. Interestingly, changes in factor endowments have played a major role in partially offsetting the contributions of the two sources.

Shantayanan Devarajan and Sherman Robinson survey the experience of the policy use of AGE models. The models have been used for assessing policies relating to international trade, public finance, agriculture, income distribution, and energy and environmental policy. The authors draw a distinction between "stylized models," which tend to be small, narrowly focused, and capture a particular mechanism through which policy influences derived outcomes, and "applied models," which are much larger, capture important institutional characteristics of the economy being modeled, and encompass a wider spectrum of issues. In stylized models, the link between policy changes and their outcomes is transparent, whereas in the applied models, the link is often difficult to see. Such lack of transparency can dissuade policy makers from using these models, even though they are based on a more realistic description of the economy and a better recognition of often

complex policy linkages. The authors list a set of desiderata for ensuring the success of the policy use of AGE models and recommend the complementary use of applied and stylized models to enhance the effectiveness of both in policy debates.

## KEY ISSUES IN APPLIED GENERAL EQUILIBRIUM MODELING

As AGE modeling has grown over the years, it has frequently displaced more conventional econometric modeling in policy analysis, but it has also encountered fresh problems. Its great strength has been its ability to provide numerical assessments of the equity and efficiency implications of micro policy change, something hard to do with conventional econometric models. On the one hand, in situations of simultaneous changes in several policies, in which interaction among policies of different countries could be significant, there is no alternative to AGE for assessing the effects of policy changes. On the other hand, many questions arise, and indeed have been raised, over the empirical plausibility of AGE model results.

These questions range from the observations that the particular equilibrium structure and functional forms used will, to a large degree, predetermine the results and that the key parameter values used (especially elasticities) are known with little certainty to the claim that there has been little or no ex post validation of model projections. When taken together with the claim that, in practice, actual models are often uneasy compromises compared to their theoretically pure parents, such questions have led some to doubt that anything of value can be found from the numerical calculations resulting from these models.

The relevant point for comparison in evaluating this work is the next best alternative and not some absolute standard devised in a mistaken analogy to the natural sciences. Policy makers find model calculations useful because for the questions they ask the only other alternative is guess-work, which is unlikely to be well informed. In contrast, well-specified AGE models are internally consistent and force anyone who is not satisfied with their results to think through the reasons for dissatisfaction. Is the source of dissatisfaction the unsatisfactory structure of the model, the values of its parameters, or the interpretation of model results? The interactive process of modeling, generating results, and analyzing the potential reasons that the results can or cannot be accepted raises the level of argument in policy process. Such a discussion avoids the pretense of providing or being able to provide definitive answers to policy questions. This is necessary if policy makers are to find AGE models a useful tool.

Nonetheless, there can be no denying that work on AGE modeling has both raised and faced many challenges and that these point the way forward for the field. Calibration inevitably implies subjective judgment by the calibrator. How is this to be squared with econometric rigor?

Ex post validation and the use of models for ex post analysis, rather than only ex ante policy evaluation, are another challenge. The claims made for the empirical

validity of or support for calibrated dynamic, stochastic, general equilibrium models of the business cycle are contentious.[3]

Elasticity parameters and the poor state of parameter estimation in empirical economics are another problem area. Statistical work in economics, following Karl Popper and Milton Friedman, still is strongly associated with hypothesis testing rather than estimation, but AGE models are dense with parameters, the values of which have to be calibrated if econometric estimates are unavailable. Often no estimates exist of required parameters, so they are guessed; or multiple estimates exist that are contradictory. In the econometric literature different estimation procedures, different data series, and different theoretical concepts are used, making it very difficult to use estimates drawn from the literature.

Another problem is the potential for misuse of models. The rather baroque structure of some of the models leads to a problem in clearly identifying the links between policy changes and their outcomes. This nontransparency leads nonmodelers even to suggest that models have been deviously constructed backward in such a way as to support and corroborate particular prior positions on an issue, and as such models are viewed by them as little more than tools of propaganda. While the modelers would no doubt dismiss such claims as verging on the hysterical, they can undermine the political legitimacy of model results. AGE models, while becoming central to policy analysis around the world, have critics as well. This poses challenges for the years ahead.

## ON TO NUMERICAL SIMULATION

AGE modeling is being used ever more widely. In economic theory the inability to obtain unambiguous general results even under fairly strong assumptions on the model's structure has led to the use of illustrative calculations based on quasi-plausible parameters. Economics is evolving like other disciplines (astrophysics, life sciences) so that the numerical representation of theoretical constraints and the resulting implications are becoming major issues.

How does a theoretical structure or model behave under plausible numerical representation and parameterizations? If theory is silent as to the sign of the effect of a change, what does the simulation suggest? Is the effect big or is it small, and by what criteria? Why do the observed sign and size of effect occur? Are these effects plausible? How are we sure there are no coding or conceptual errors? Can results be replicated? How robust are they?

[3] There is a deeper problem with the use of estimated parameters from the literature on AGE models. Many estimated parameters, including some of the elasticities, are not what Robert Lucas calls "deep" – invariant parameters of tastes and technology. This means that their estimates are subject to the Lucas critique that they are policy-regime specific, so that values estimated with data from one regime cannot be used for analysis of data from a different regime. Even if the data are treated as representing an equilibrium, the restrictions on parameters that an equilibrium implies are rarely imposed in estimation. The paper by Heckman, Matzkin, and Nesheim in this volume stresses the important role in estimation played by such conditions.

These then become the primary issues for the emerging field of computational economics – which is loosely related to the field of AGE modeling – a field that is slowly becoming dominant in such areas as macroeconomics and will, in our view, increasingly engulf all of economics. The goal is to use numerical methods to assess the implications of analytical structures, both in policy and analysis and for understanding the world around us. Because this work is necessarily subjective in design and execution, the credibility of modelers is key. Their ability to communicate what they have done, why they have done it, and what they conclude is absolutely central.

Some years ago, Peter Wiles (1962) aptly characterized the then debate on computational methods in economics as the unsolved problem of "the perfect computation of perfect competition." Herbert Scarf's algorithm and thesis advising solved it and achieved in large measure the perfection that Wiles imagined. The challenge for the next generation of AGE modelers is to take Scarf's achievement one stage further. We are confident that the chapters in this volume will set the stage for meeting it.

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