

Contents

List of exercises	xi
Preface to the Series	xxv
Preface	xxix
1 Vectors	1
1.1 Real vectors	4
1.2 Complex vectors	11
2 Matrices	15
2.1 Real matrices	19
2.2 Complex matrices	39
3 Vector spaces	43
3.1 Complex and real vector spaces	47
3.2 Inner-product space	61
3.3 Hilbert space	67
4 Rank, inverse, and determinant	73
4.1 Rank	75
4.2 Inverse	83
4.3 Determinant	87
5 Partitioned matrices	97
5.1 Basic results and multiplication relations	98
5.2 Inverses	103
5.3 Determinants	109
5.4 Rank (in)equalities	119
5.5 The sweep operator	126
6 Systems of equations	131
6.1 Elementary matrices	132
6.2 Echelon matrices	137

viii	<i>Contents</i>	
	6.3 Gaussian elimination	143
	6.4 Homogeneous equations	148
	6.5 Nonhomogeneous equations	151
7	Eigenvalues, eigenvectors, and factorizations	155
	7.1 Eigenvalues and eigenvectors	158
	7.2 Symmetric matrices	175
	7.3 Some results for triangular matrices	182
	7.4 Schur’s decomposition theorem and its consequences	187
	7.5 Jordan’s decomposition theorem	192
	7.6 Jordan chains and generalized eigenvectors	201
8	Positive (semi)definite and idempotent matrices	209
	8.1 Positive (semi)definite matrices	211
	8.2 Partitioning and positive (semi)definite matrices	228
	8.3 Idempotent matrices	231
9	Matrix functions	243
	9.1 Simple functions	246
	9.2 Jordan representation	255
	9.3 Matrix-polynomial representation	265
10	Kronecker product, vec-operator, and Moore-Penrose inverse	273
	10.1 The Kronecker product	274
	10.2 The vec-operator	281
	10.3 The Moore-Penrose inverse	284
	10.4 Linear vector and matrix equations	292
	10.5 The generalized inverse	295
11	Patterned matrices: commutation- and duplication matrix	299
	11.1 The commutation matrix	300
	11.2 The symmetrizer matrix	307
	11.3 The vech-operator and the duplication matrix	311
	11.4 Linear structures	318
12	Matrix inequalities	321
	12.1 Cauchy-Schwarz type inequalities	322
	12.2 Positive (semi)definite matrix inequalities	325
	12.3 Inequalities derived from the Schur complement	341
	12.4 Inequalities concerning eigenvalues	343
13	Matrix calculus	351
	13.1 Basic properties of differentials	355
	13.2 Scalar functions	356
	13.3 Vector functions	360
	13.4 Matrix functions	361
	13.5 The inverse	364
	13.6 Exponential and logarithm	368
	13.7 The determinant	369

<i>Contents</i>	<i>ix</i>
13.8 Jacobians	373
13.9 Sensitivity analysis in regression models	375
13.10 The Hessian matrix	378
13.11 Least squares and best linear unbiased estimation	382
13.12 Maximum likelihood estimation	387
13.13 Inequalities and equalities	391
Appendix A: Some mathematical tools	397
A.1 Some methods of indirect proof	397
A.2 Primer on complex numbers and polynomials	398
A.3 Series expansions	401
A.3.1 Sequences and limits	402
A.3.2 Convergence of series	403
A.3.3 Special series	404
A.3.4 Expansions of functions	407
A.3.5 Multiple series, products, and their relation	408
A.4 Further calculus	409
A.4.1 Linear difference equations	409
A.4.2 Convexity	410
A.4.3 Constrained optimization	410
Appendix B: Notation	415
B.1 Vectors and matrices	415
B.2 Mathematical symbols, functions, and operators	418
Bibliography	423
Index	426