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978-0-521-82284-8 - Quantum Field Theory in Condensed Matter Physics

Alexei M. Tsvelik

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Quantum Field Theory in Condensed Matter Physics

This book is a course in modern quantum field theory as seen through the eyes of a theorist working in condensed matter physics. It contains a gentle introduction to the subject and can therefore be used even by graduate students. The introductory parts include a derivation of the path integral representation, Feynman diagrams and elements of the theory of metals including a discussion of Landau Fermi liquid theory. In later chapters the discussion gradually turns to more advanced methods used in the theory of strongly correlated systems. The book contains a thorough exposition of such nonperturbative techniques as $1/N$ -expansion, bosonization (Abelian and non-Abelian), conformal field theory and theory of integrable systems. The book is intended for graduate students, postdoctoral associates and independent researchers working in condensed matter physics.

ALEXEI TSVELIK was born in 1954 in Samara, Russia, graduated from an elite mathematical school and then from Moscow Physical Technical Institute (1977). He defended his PhD in theoretical physics in 1980 (the subject was heavy fermion metals). His most important collaborative work (with Wiegmann on the application of Bethe ansatz to models of magnetic impurities) started in 1980. The summary of this work was published as a review article in *Advances in Physics* in 1983. During the years 1983–89 Alexei Tsvelik worked at the Landau Institute for Theoretical Physics. After holding several temporary appointments in the USA during the years 1989–92, he settled in Oxford, where he spent nine years. Since 2001 Alexei Tsvelik has held a tenured research appointment at Brookhaven National Laboratory. The main area of his research is strongly correlated systems (with a view of application to condensed matter physics). He is an author or co-author of approximately 120 papers and two books. His most important papers include papers on the integrable models of magnetic impurities, papers on low-dimensional spin liquids and papers on applications of conformal field theory to systems with disorder. Alexei Tsvelik has had nine graduate students of whom seven have remained in physics.

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To my father

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Preface to the first edition

The objective of this book is to familiarize the reader with the recent achievements of quantum field theory (henceforth abbreviated as QFT). The book is oriented primarily towards condensed matter physicists but, I hope, can be of some interest to physicists in other fields. In the last fifteen years QFT has advanced greatly and changed its language and style. Alas, the fruits of this rapid progress are still unavailable to the vast democratic majority of graduate students, postdoctoral fellows, and even those senior researchers who have not participated directly in this change. This cultural gap is a great obstacle to the communication of ideas in the condensed matter community. The only way to reduce this is to have as many books covering these new achievements as possible. A few good books already exist; these are cited in the select bibliography at the end of the book. Having studied them I found, however, that there was still room for my humble contribution. In the process of writing I have tried to keep things as simple as possible; the amount of formalism is reduced to a minimum. Again, in order to make life easier for the newcomer, I begin the discussion with such traditional subjects as path integrals and Feynman diagrams. It is assumed, however, that the reader is already familiar with these subjects and the corresponding chapters are intended to refresh the memory. I would recommend those who are just starting their research in this area to read the first chapters in parallel with some introductory course in QFT. There are plenty of such courses, including the evergreen book by Abrikosov, Gorkov and Dzyaloshinsky. I was trained with this book and thoroughly recommend it.

Why study quantum field theory? For a condensed matter theorist as, I believe, for other physicists, there are several reasons for studying this discipline. The first is that QFT provides some wonderful and powerful tools for our research. The results achieved with these tools are innumerable; knowledge of their secrets is a key to success for any decent theorist. The second reason is that these tools are also very elegant and beautiful. This makes the process of scientific research very pleasant indeed. I do not think that this is an accidental coincidence; it is my strong belief that aesthetic criteria are as important in science as empirical ones. Beauty and truth cannot be separated, because ‘beauty is truth realized’ (Vladimir Solovyev). The history of science strongly supports this belief: all great physical theories are at the same time beautiful. Einstein, for example, openly admitted that ideas of beauty played a very important role in his formulation of the theory of general relativity, for which any experimental support had remained minimal for many years. Einstein is by no

means alone; the reader is advised to read the philosophical essays of Werner Heisenberg, whose authority in the area of physics is hard to deny. Aesthetics deals with forms; it is not therefore suprising that a smack of geometry is felt strongly in modern QFT: for example, the idea that a vacuum, being an apparently empty space, has a certain symmetry, i.e. has a geometric figure associated with it. In what follows we shall have more than one chance to discuss this particular topic and to appreciate the fact that geometrical constructions play a major role in the behaviour of physical models.

The third reason for studying QFT is related to the first and the second. QFT has the power of *universality*. Its language plays the same unifying role in our times as Latin played in the times of Newton and Leibniz. Its knowledge is the equivalent of literacy. This is not an exaggeration: equations of QFT describe phase transitions in magnetic metals and in the early universe, the behaviour of quarks and fluctuations of cell membranes; in this language one can describe equally well both classical and quantum systems. The latter feature is especially important. From the very beginning I shall make it clear that from the point of view of calculations, there is no difference between quantum field theory and classical statistical mechanics. Both these disciplines can be discussed within the same formalism. Therefore everywhere below I shall unify quantum field theory and statistical mechanics under the same abbreviation of QFT. This language helps one

To see a world in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
And eternity in an hour.*

I hope that by now the reader is sufficiently inspired for the hard work ahead. Therefore I switch to prose. Let me now discuss the content of the book. One of its goals is to help the reader to solve future problems in condensed matter physics. These are more difficult to deal with than past problems, all the easy ones have already been solved. What remains is difficult, but is interesting nevertheless. The most interesting, important and complicated problems in QFT are those concerning strongly interacting systems. Indeed, most of the progress over the past fifteen years has been in this area. One widely known and related problem is that of quark confinement in quantum chromodynamics (QCD). This still remains unresolved, as far as I am aware. A less known example is the problem of strongly correlated electrons in metals near the metal–insulator transition. The latter problem is closely related to the problem of high temperature superconductivity. Problems with the strong interaction cannot be solved by traditional methods, which are mostly related to perturbation theory. This does not mean, however, that it is not necessary to learn the traditional methods. On the contrary, complicated problems cannot be approached without a thorough knowledge of more simple ones. Therefore Part I of the book is devoted to such traditional methods as the path integral formulation of QFT and Feynman diagram expansion. It is not supposed, however, that the reader will learn these methods from this book. As I have

* William Blake, *Auguries of Innocence*.

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said before, there are many good books which discuss the traditional methods, and it is not the purpose of Part I to be a substitute for them, but rather to recall what the reader has learnt elsewhere. Therefore discussion of the traditional methods is rather brief, and is targeted primarily at the aspects of these methods which are relevant to nonperturbative applications.

The general strategy of the book is to show how the strong interaction arises in various parts of QFT. I do not discuss in detail all the existing condensed matter theories where it occurs; the theories of localization and quantum Hall effect are omitted and the theory of heavy fermion materials is discussed only very briefly. Well, one cannot embrace the unembraceable! Though I do not discuss all the relevant physical models, I do discuss all the possible scenarios of renormalization: there are only three of them. First, it is possible that the interactions are large at the level of a bare many-body Hamiltonian, but effectively vanish for the low energy excitations. This takes place in quantum electrodynamics in $(3 + 1)$ dimensions and in Fermi liquids, where scattering of quasi-particles on the Fermi surface changes only their phase (forward scattering). Another possibility is that the interactions, being weak at the bare level, grow stronger for small energies, introducing profound changes in the low energy sector. This type of behaviour is described by so-called ‘asymptotically free’ theories; among these are QCD, the theories describing scattering of conducting electrons on magnetic impurities in metals (the Anderson and the s-d models, in particular), models of two-dimensional magnets, and many others. The third scenario leads us to critical behaviour. In this case the interactions between low energy excitations remain finite. Such situations occur at the point of a second-order phase transition. The past few years have been marked by great achievements in the theory of two-dimensional second-order phase transitions. A whole new discipline has appeared, known as conformal field theory, which provides us with a potentially complete description of all types of possible critical points in two dimensions. The classification covers two-dimensional theories at a transition point and those quantum $(1 + 1)$ -dimensional theories which have a critical point at $T = 0$ (the spin $S = 1/2$ Heisenberg model is a good example of the latter).

In the first part of the book I concentrate on formal methods; at several points I discuss the path integral formulation of QFT and describe the perturbation expansion in the form of Feynman diagrams. There is not much ‘physics’ here; I choose a simple model (the $O(N)$ -symmetric vector model) to illustrate the formal procedures and do not indulge in discussions of the physical meaning of the results. As I have already said, it is highly desirable that the reader who is unfamiliar with this material should read this part in parallel with some textbook on Feynman diagrams. The second part is less dry; here I discuss some miscellaneous and relatively simple applications. One of them is particularly important: it is the electrodynamics of normal metals where on a relatively simple level we can discuss violations of the Landau Fermi liquid theory. In order to appreciate this part, the reader should know what is violated, i.e. be familiar with the Landau theory itself. Again, I do not know a better book to read for this purpose than the book by Abrikosov, Gorkov and Dzyaloshinsky. The real fun starts in the third and the fourth parts, which are fully devoted to nonperturbative methods. I hope you enjoy them!

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Finally, those who are familiar with my own research will perhaps be surprised by the absence in this book of exact solutions and the Bethe ansatz. This is not because I do not like these methods any more, but because I do not consider them to be a part of the *minimal* body of knowledge necessary for any theoretician working in the field.

Alexei Tsvelik
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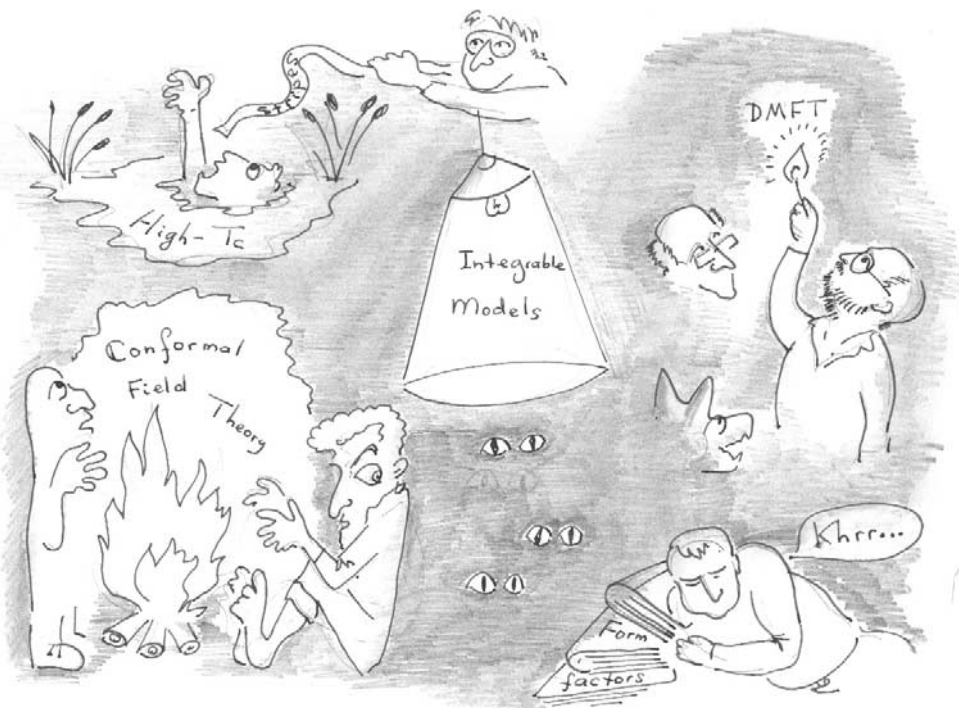
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Preface to the second edition

Though it was quite beyond my original intentions to write a textbook, the book is often used to teach graduate students. To alleviate their misery I decided to extend the introductory chapters and spend more time discussing such topics as the equivalence of quantum mechanics and classical statistical mechanics. A separate chapter about Landau Fermi liquid theory is introduced. I still do not think that the book is fully suitable as a graduate textbook, but if people want to use it this way, I do not object.

Almost 10 years have passed since I began my work on the first edition. The use of field theoretical methods has extended enormously since then, making the task of rewriting the book very difficult. I no longer feel myself capable of presenting a brief course containing the ‘minimal body of knowledge necessary for any theoretician working in the field’. I strongly feel that such a body of knowledge should include not only general ideas, what is usually called ‘physics’, but also techniques, even technical tricks. Without this common background we shall not be able to maintain high standards of our profession and the fragmentation of our community will continue further. However, the best I can do is to include the material I can explain well and to mention briefly the material which I deem worthy of attention. In particular, I decided to include exact solutions and the Bethe ansatz. It was excluded from the first edition as being too esoteric, but now the astonishing new progress in calculations of correlation functions justifies its inclusion in the core text. I think that this progress opens new exciting opportunities for the field, but the community has not yet woken up to the change. The chapters about the two-dimensional Ising model are extended. Here again the community does not fully grasp the importance of this model and of the concepts related to it. For the same reason I extended the chapters devoted to the Wess–Zumino–Novikov–Witten model.

Alexei Tsvelik
Brookhaven, 2002



Fragmentation of the community.

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I am infinitely grateful to my friends and colleagues Alexander Nersesyan, Andrei Chubukov, Fabian Essler, Alexander Gogolin and Joe Bhaseen for support and advice. I am also grateful to my new colleagues at Brookhaven National Laboratory, especially to Doon Gibbs and Peter Johnson, who made my transition to the USA so smooth and pleasant. I also acknowledge support from US DOE under contract number DE-AC02-98 CH 10886.