

ADVANCED ASTROPHYSICS

This book develops the basic underlying physics required for a fuller, richer understanding of the science of astrophysics and the important astronomical phenomena it describes. The Cosmos manifests phenomena in which physics can appear in its most extreme, and therefore more insightful, forms. A proper understanding of phenomena such as black holes, quasars and extrasolar planets requires that we understand the physics that underlies all of astrophysics. Consequently, developing astrophysical concepts from fundamental physics has the potential to achieve two goals: to derive a better understanding of astrophysical phenomena from first principles and to illuminate the physics from which the astrophysics is developed. To that end, astrophysical topics are grouped according to the relevant areas of physics. The book is ideal as a text for graduate students and as a reference for established researchers.

The author obtained his PhD in 1984 from the University of Toronto where he earned the Royal Astronomical Society of Canada Gold Medal for academic excellence. After a brief postdoctoral stint at the University of British Columbia, he joined the faculty at the University of New Mexico where he pursued his interests in radio astronomy. He has been teaching for the past 17 years, earning an “excellence in teaching” award for the graduate courses on which this book is based. Dr. Duric has over 100 scientific publications and has authored and/or edited five books. In addition, he has developed a number of online classes, including a completely interactive, web-based freshman astronomy course. He is the recipient of the “Regent’s Fellowship”, the highest honour that UNM bestows on its faculty. His research has taken him around the world to over a dozen countries, accounting in part, for the global perspective that characterizes his book. Dr. Duric is a member of the American Astronomical Society and the Canadian Astronomical Society.



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NEB DURIC

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Contents

<i>Preface</i>	<i>page xiii</i>
Part I Classical mechanics	1
1 Orbital mechanics	3
1.1 Universal gravitation	3
1.1.1 Center of mass	3
1.1.2 Reduced mass	4
1.2 Kepler's laws	5
1.2.1 Planetary orbits	9
1.3 Binary stars	11
1.3.1 Visual binaries	11
1.3.2 The apparent orbit	11
1.3.3 The true orbit	12
1.3.4 Determining the orbital elements	14
1.3.5 Spectroscopic binaries	14
1.3.6 The mass function	17
1.3.7 Summary of binary star studies	18
1.3.8 Mass–luminosity relation	19
1.4 Extrasolar planets	20
1.4.1 The astrometric method	21
1.4.2 The radial velocity method	22
1.4.3 The transit method	24
1.5 References	25
1.6 Further reading	25
2 Galaxy dynamics	26
2.1 Potentials of arbitrary matter distributions	26
2.2 Dynamics of thin disks	27

2.3	Rotation curves of disk galaxies	30
2.3.1	Rotation curves of real spiral galaxies	31
2.4	N -body gravitational systems	34
2.4.1	Equation of motion	34
2.4.2	The Virial theorem	35
2.4.3	Clusters of galaxies	36
2.5	References	39
2.6	Further reading	39
3	Cosmic expansion and large scale structure	40
3.1	The expansion of the Universe	40
3.1.1	The cosmological constant	42
3.2	Large-scale cosmic structure	45
3.2.1	Overview	45
3.2.2	Correlation functions of galaxies	46
3.2.3	Dark matter and large-scale structure	47
3.2.4	Hot and cold dark matter	51
3.2.5	The Jeans' mass and gravitational stability	52
3.2.6	Possible models of structure formation	54
3.3	References	55
3.4	Further reading	55
	Part II Statistical mechanics	57
4	Overview of statistical mechanics	59
4.1	Thermodynamics	59
4.2	Classical statistical mechanics	61
4.3	Quantum statistical mechanics	63
4.3.1	Bose–Einstein statistics	64
4.3.2	Fermi–Dirac statistics	64
4.4	Photon distribution function	65
4.5	Thermodynamic equilibrium	66
4.6	Further reading	68
5	The early Universe	69
5.1	The 3 K background radiation	69
5.1.1	History of the background radiation	69
5.1.2	Evolution of energy density	70
5.2	Galaxy formation	71
5.3	Local cosmology and nucleosynthesis	72
5.3.1	Overview	72
5.3.2	Primordial helium	73
5.4	Reaction rates	75
5.4.1	Introduction	75

5.4.2	Barrier penetration	77
5.4.3	Estimating reaction rates	81
5.4.4	Destruction of D	82
5.4.5	Formation of D	82
5.4.6	Formation of ^4He	83
5.5	Particle equilibria in the early Universe	83
5.5.1	Overview	83
5.5.2	Chemical equilibrium	86
5.5.3	The early Universe	87
5.5.4	The neutron–proton ratio	87
5.5.5	Reaction freeze-out	88
5.5.6	Reaction timescale	90
5.5.7	Formation of deuterium	91
5.6	Further reading	93
6	Stellar structure and compact stars	94
6.1	Hydrostatic equilibrium	94
6.2	Fermion degeneracy	97
6.2.1	White dwarf equation of state	100
6.2.2	Mass–radius relation for white dwarfs	100
6.3	Internal structure of white dwarfs	100
6.3.1	Relationship between pressure and energy density	101
6.3.2	Relating electron number density to the mass density	104
6.3.3	Other sources of pressure	105
6.3.4	Equation of state	105
6.3.5	Internal structure of white dwarfs	105
6.3.6	Estimating the radius and mass of a white dwarf	107
6.4	Stability of compact stars	109
6.4.1	Total energy	109
6.4.2	Electron capture	110
6.4.3	Maximum density	111
6.5	Structure of neutron stars	114
6.5.1	Overview	114
6.5.2	Liquid layer	115
6.5.3	The crust	117
6.5.4	The core	119
6.6	Pulsars	119
6.7	Further reading	121

Part III Electromagnetism	123
7 Radiation from accelerating charges	125
7.1 The Lienard–Wiechert potential	125
7.1.1 Scalar and vector potentials	125
7.1.2 Green’s function solution	126
7.1.3 The L–W potentials	127
7.2 Electric and magnetic fields of a moving charge	128
7.2.1 Moving charge at constant velocity	129
7.2.2 Radiation from accelerating charges – the far zone	131
7.2.3 Angular distribution of radiation	131
7.2.4 Total emitted power	133
7.3 Further reading	134
8 Bremsstrahlung and synchrotron radiation	135
8.1 Bremsstrahlung	135
8.1.1 Single particle collisions	135
8.1.2 Radiation from an ensemble of particles	137
8.2 Synchrotron radiation	138
8.2.1 Total power	139
8.2.2 The received spectrum	140
8.2.3 Spectrum of a power-law energy distribution	141
8.3 Further reading	143
9 High energy processes in astrophysics	144
9.1 Neutron stars	144
9.2 Supernova remnants	145
9.2.1 Particle acceleration	145
9.3 Radio galaxies	148
9.4 Galactic X-ray sources	152
9.4.1 The energy source	153
9.4.2 Maximum luminosity/Eddington limit	153
9.4.3 Characteristic temperature	154
9.4.4 Mass transfer	154
9.5 Accretion disks	154
9.5.1 Disk hydrodynamics	156
9.5.2 The emission spectrum of the disk	157
9.6 Pulsars revisited	159
9.6.1 The radiation field	160
9.6.2 Radiated power	161
9.6.3 The Braking Index	162
9.6.4 The static magnetic field	162
9.6.5 The static electric field	162

9.7	Reference	163
9.8	Further reading	163
10	Electromagnetic wave propagation	164
10.1	EM waves in an un-magnetized plasma	165
10.1.1	Dispersion measure	166
10.2	EM waves in a magnetized medium	167
10.2.1	Rotation measure	170
10.3	Reference	172
10.4	Further reading	172
	Part IV Quantum mechanics	173
11	The hydrogen atom	175
11.1	Structure of the hydrogen atom	175
11.1.1	Case 1 $r \rightarrow \infty$	177
11.1.2	Case 2 $r \rightarrow 0$	177
11.1.3	What about the in-between?	178
11.1.4	Normalizing $R(r)$	179
11.2	Total wave function	181
11.3	Probability functions	181
11.4	Energy eigenstates and transitions	185
11.5	Further reading	185
12	The interaction of radiation with matter	187
12.1	Non-relativistic treatment	187
12.2	Single particle Hamiltonian	188
12.3	Separation of static and radiation fields	189
12.3.1	Relative importance of H_0 , H_1 and H_2	189
12.4	Radiative transitions	190
12.4.1	Semi-classical approach	190
12.4.2	The Hamiltonian of the radiation field	191
12.4.3	The perturbation Hamiltonian	192
12.4.4	Time-dependent perturbation theory	193
12.5	Absorption of photons	194
12.5.1	Absorption cross-sections	196
12.5.2	Dipole transition probability	197
12.5.3	Bound-bound absorption cross-section	198
12.6	Spontaneous emission	199
12.7	Photoionization	200
12.7.1	Bound-free cross-sections	202
12.8	Selection rules	203
12.8.1	Dipole selection rules	204
12.8.2	Electric quadrupole transitions	205

12.9	Numerical evaluation of transition probabilities	206
12.9.1	The Lyman α transition	207
12.9.2	Bound–free absorption cross-section	209
12.10	HII regions	210
12.10.1	Ionizing stars	211
12.11	Ionization of a pure hydrogen nebula	212
12.11.1	Radius of HII region	216
12.12	Quasars and the Lyman α forest	216
12.12.1	Correlation studies	218
12.12.2	Column density of the HI responsible for the Ly- α forest	220
12.13	Reference	220
12.14	Further reading	221
13	Atomic fine structure lines	222
13.1	Electron spin	222
13.1.1	Relativistic Hamiltonian	222
13.2	Dirac’s postulate	223
13.2.1	The Dirac equation	224
13.2.2	Free particle at rest	224
13.2.3	Non-relativistic limit of Dirac’s equation	225
13.3	Radiative transitions involving spin	227
13.3.1	Zeeman effect	228
13.4	Relativistic correction with $A = 0$	228
13.5	Atomic fine structure	229
13.5.1	Spin–orbit interaction	230
13.5.2	Time-independent perturbation theory	230
13.5.3	The jm representation	231
13.5.4	Solution for E_{SO}	232
13.6	Further reading	234
14	Atomic hyperfine lines	235
14.1	The 21 cm line of hydrogen	235
14.1.1	Transition rate	238
14.1.2	The 21 cm line profile	239
14.2	The Doppler effect	240
14.2.1	Doppler broadening of the 21 cm line	241
14.3	Neutral hydrogen in galaxies	242
14.3.1	Equation of transfer for HI emission	242
14.3.2	Emission or absorption?	244

14.4	Measuring HI in external galaxies	244
14.4.1	Integral properties of galaxies	245
14.4.2	Kinematics of the HI	245
14.5	Probing galactic mass distributions	246
14.5.1	HI rotation curves	247
14.6	References	253
14.7	Further reading	253
15	Transitions involving multi-electron atoms	254
15.1	Symmetry of multi-particle wave functions	254
15.2	The helium atom	255
15.2.1	The ground state	255
15.2.2	Lowest excited states	257
15.2.3	Summary	258
15.3	Many-electron atoms	259
15.3.1	The Hartree–Fock procedure	260
15.4	Forbidden lines in astrophysics	261
15.4.1	Collisional equilibria	261
15.4.2	Line emission and cooling of nebulae	263
15.4.3	Statistical equilibrium for N levels	265
15.4.4	OIII lines as probes of temperature	266
15.4.5	Line ratios as density probes	269
15.4.6	Observations of nebulae	270
15.4.7	Observations of active galactic nuclei	270
15.5	Further reading	272
16	Molecular lines in astrophysics	273
16.1	Diatomic molecules	273
16.1.1	Inter-nuclear potential	273
16.1.2	Electronic transitions	273
16.1.3	Vibrational transitions	274
16.1.4	Rotational transitions	274
16.1.5	Summary	275
16.2	Diatomic molecules with two valence electrons	275
16.2.1	The Born–Oppenheimer Approximation	276
16.3	Translational and internal degrees of freedom	278
16.3.1	Vibrations and rotations	279
16.3.2	Vibrations – harmonic oscillator approximation	280
16.4	Dipole transition probability	281
16.4.1	Pure rotational spectra	282
16.5	Transitions between vibrational levels	283

16.6	Transitions between electronic levels	284
16.7	The H ₂ molecule	285
16.8	The CO molecule	287
16.9	Other molecules	290
16.10	Life in the Universe	291
	16.10.1 The building blocks of life	291
	16.10.2 Communicating with other civilizations	293
16.11	Further reading	293
	<i>Index</i>	294

Preface

Astrophysics strives to describe the Universe through the application of fundamental physics. The Cosmos manifests phenomena in which the physics can appear in its most extreme, and therefore more insightful, forms. Consequently, developing astrophysical concepts from fundamental physics has the potential to achieve two goals: to derive a better understanding of astrophysical phenomena from first principles, and to illuminate the physics from which the astrophysics is developed. To that end, astrophysical topics are grouped, in this book, according to the relevant areas of physics. For example, the derivation of the laws of orbital motion, used in the detection of extrasolar planets, takes place in the classical mechanics part of the book while the derivation of transition rates for the 21 cm neutral hydrogen line, used to probe galaxy kinematics, is performed in the quantum mechanics part. The book could serve as a text for graduate students and as a reference for established researchers.

The content of this book is based on the material used by the author in support of advanced astrophysics courses taught at the University of New Mexico. The intended audience consists of graduate students and senior undergraduates pursuing degrees in physics and/or astrophysics. Perhaps the most directly relevant demographic is the combined Physics and Astronomy departments. These departments tend to emphasize the fundamental physics regardless of the research track pursued by the student. In many cases a separate astrophysics degree is not an option. In these departments (such as the author's) all students must pass the same physics comprehensive examination. Consequently, students must be well prepared in fundamental physics both from the points of view of course work as well as research. In the latter case a strong physics foundation is very helpful in developing thesis topics to an acceptable level in a physics-dominated department. This book is specifically aimed at those departments.

The department of Physics and Astronomy at the University of New Mexico requires its graduate students to take the physics comprehensive exam. Courses

based on the material in this book have helped astrophysics and physics students prepare for these exams. I attribute this benefit to the fact that the astrophysical topics provide interesting and insightful manifestations of the fundamental physics, of which, the students previously may have had only a theoretical knowledge. I therefore expect the book to impact the physics as well as the astrophysics students in the mixed departments. I also expect that graduate students in physics-only and astronomy-only departments may choose to use this book to hone their research skills. Targeting junior/senior undergraduates is also possible in schools where the science curricula are robust.

Multi-disciplinary and cross-disciplinary investigations are playing an increasingly important role in scientific research. The cross-over of particle physicists into cosmology, and the establishment of the field of astroparticle physics, is just one manifestation of the growing overlap between physics and astrophysics disciplines. The emphasis on the linkage of fundamental physics and astrophysics makes this book potentially useful as a reference to physics and astrophysics researchers who wish to broaden their research base.

The author acknowledges the help of Dr. Rich Epstein (Los Alamos National Laboratory) who co-taught the first course in the series of courses that have led to the development of the material used in this book. The cooperation and help of the many students who have taken these courses has been instrumental in identifying many typos and inconsistencies in the course material. Finally, the author acknowledges the help and support of the department of physics and astronomy at UNM and the patience and encouragement of family and friends in this endeavor.