Introduction

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In the late 1950s, when Gribov, then a young researcher at the Ioffe Physico-technical Institute, became interested in the physics of strong hadron interactions, there was no consistent picture of high energy scattering processes, not to mention *a theory.* Apart from the Pomeranchuk theorem – an asymptotic equality of particle and antiparticle cross sections [1] – not much was theoretically understood about processes at high energies.

Gribov's 1961 paper 'Asymptotic behaviour of the scattering amplitude at high energies' (submitted to *Nucl. Phys.* on June 28, 1960) in which he proved an inconsistency of the *black disk model* of diffractive hadron–hadron scattering may be considered a first building block of the modern theory of high energy particle interactions [2].

Gribov's use of the so-called double dispersion representation for the scattering amplitude, suggested by S. Mandelstam back in 1958 [3], demonstrated the combined power of the general principles of relativistic quantum theory – unitarity (conservation of probability), analyticity (causality) and the relativistic nature (crossing symmetry) – as applied to high energy interactions.

The then-standard black disk model viewed a hadron as an object with a finite interaction radius that did not depend on collision energy, and employed for the imaginary part of the scattering amplitude the factorized expression

$$A_1(s,t) = s\,f(t). \tag{0.1}$$

By studying the analytic properties in the cross-channels, Gribov showed that the model (0.1) for diffraction in the physical region of s-channel scattering contradicts the unitarity relation for partial waves in the crossing t-channel. To solve the puzzle, he suggested the behaviour of the amplitude (for large s and finite t) in the general form

$$A_1(s,t) = s^{q(t)}\,B_t(\ln s)\,, \tag{0.2}$$

where B_t is a slow (non-exponential) function of $\ln s$ (decreasing fast with t), and $q(0) = 1$ ensures the approximate constancy of the total cross section, $\sigma^{\rm tot}(s) \simeq$ const.

In this first paper Gribov analysed the constant exponent, $q(t) = 1$, and proved that the cross section in this case has to decrease at high energies, $B_t(\ln s) < 1/\ln s$, to be consistent with the t-channel unitarity. He remarked on the possibility $q(t) \neq \text{const}$ as 'extremely unlikely' since, considering the t-dependence of the scattering amplitude, this would correspond to a strange picture of the radius of a hadron infinitely increasing with energy. He decided to 'postpone the treatment of such rapidly chan ging functions until a more detailed investigation is carried out'.

He published the results of such an investigation the next year in the letter to *ZhETF* 'Possible asymptotic behaviour of elastic scattering'. In his letter Gribov discussed the asymptotic behaviour 'which in spite of having a few unusual features is theoretically feasible and does not contradict the experimental data' [4]. Gribov was already aware of the finding by T. Regge [5] that in non-relativistic quantum mechanics

$$A(s,t) \propto t^{\ell(s)}, \tag{0.3}$$

in the unphysical region $|t| \gg s$ (corresponding to large imaginary scattering angles $\cos\Theta \to \infty$), where $\ell(s)$ is the position of the pole of the partial wave f_ℓ in the complex plane of the orbital momentum ℓ.

T. Regge found that the poles of the amplitude in the complex ℓ-plane were intimately related with bound states/resonances. It is this aspect of the Regge behaviour that initially attracted the most attention:

> S. Mandelstam has suggested and emphasized repeatedly since 1960 that the Regge behavior would permit a simple description of dynamical states (private discussions). Similar remarks have been made by R. Blankenbecker and M.L. Goldberger and by K. Wilson (quoted from [6]).

Gribov learned about the Regge results from a paper by G. Chew and S. Frautschi [7] which still advocated the wrong black disk diffraction model (0.1) but contained a *footnote* describing the main Regge findings.

The structure of the Regge amplitude (0.3) motivated Gribov to return to the consideration of the case of the t-dependent exponent in his general high energy ansatz (0.2) that was dictated by t-channel unitarity.

By then M. Froissart had already proved his famous theorem that limits the asymptotic behaviour of the total cross sections [8],

$$\sigma^{\text{tot}} \propto s^{-1}\,|A_1(s,0)| < \text{const}\cdot\ln^2 s\,. \tag{0.4}$$

Thus, having accepted $\ell(0) = 1$ for the rightmost pole in the ℓ-plane as the condition 'that the strongest possible interaction is realized', Gribov formulated 'the main properties of such an asymptotic scattering behaviour':

- the total interaction cross section is constant at high energies,

- the elastic cross section tends to zero as $1/\ln s$,
- the scattering amplitude is essentially imaginary,
- the significant region of momentum transfer in elastic scattering shrinks with increasing energy, $\sqrt{-t} \propto (\ln s)^{-1/2}$.

He also analysed the s-channel partial waves to show that for small impact parameters $\rho < R$ their amplitudes fall as $1/\ln s$, while the interaction radius R increases with energy as $\rho \propto \sqrt{\ln s}$. He concluded:

> this behaviour means that the particles become grey with respect to high energy interaction, but increase in size, so that the total cross section remains constant.

These were the key features of what has become known as the 'Regge theory' of strong interactions at high energies. On the opposite side of the Iron Curtain, the basic properties of the Regge pole picture of forward/backward scattering were formulated half a year later by G. Chew and S. Frautschi in [9]. In particular, they suggested 'the possibility that the recently discovered ρ meson is associated with a Regge pole whose internal quantum numbers are those of an $I = 1$ two-pion configuration', and conjectured the universal high energy behaviour of backward $\pi^+\pi^0$, K^+K^0 and pn scattering due to ρ–reggeon exchange. G. Chew and S. Frautschi also stressed that the hypothetical Regge pole with $\alpha(0) = 1$ responsible for forward scattering possesses quantum numbers of the *vacuum*.

Dominance of the vacuum pole automatically satisfies the Pomeranchuk theorem. The name 'pomeron' for this vacuum pole was coined by Murray Gell-Mann, who referred to Geoffrey Chew as an inventor.

Shrinkage of the diffractive peak was predicted, and was experimentally verified at particle accelerator experiments in Russia (IHEP, Serpukhov), Switzerland (CERN) and the US (FNAL, Chicago), as were the general relations between the cross sections of different processes that followed from the Gribov factorization theorem [10].

In non-relativistic quantum mechanics the interaction Hamiltonian allows for scattering partial waves to be considered as analytic functions of complex angular momentum ℓ (provided the interaction potential is analytic in r).

Gribov's paper 'Partial waves with complex orbital angular momenta and the asymptotic behaviour of the scattering amplitude' showed that the partial waves with complex angular momenta can be introduced in a relativistic theory as well, on the basis of the Mandelstam double dispersion representation. Here it is the *unitarity in the crossing channel* that replaces Hamiltonian dynamics and leads to analyticity of the partial waves in ℓ. The corresponding construction is known as the 'Gribov–Froissart projection' [11].

A few months later Gribov demonstrated that the simplest (two-particle) t-channel unitarity condition indeed generates the moving *pole* singularities in the complex ℓ-plane. This was the *proof* of the Regge hypothesis in relativistic theory [12].

The 'Regge trajectories' $\alpha(t)$ combine hadrons into families: $s_h = \alpha(m_h^2)$, where s_h and m_h are the spin and the mass of a hadron (hadronic resonance) with given quantum numbers (baryon number, isotopic spin, strangeness, etc.) [9]. Moreover, at negative values of t, that is in the physical region of the s-channel, the very same function $\alpha(t)$ determines the scattering amplitude, according to (0.2). It looks *as if* high energy scattering were due to t-channel exchange of a 'particle' with spin $\alpha(t)$ that varies with momentum transfer t – the 'reggeon'.

Thus, the high energy behaviour of the scattering process $a + b \to c + d$ is linked with the spectrum of excitations (resonances) of low-energy scattering in the dual channel, $a + \bar{c} \to \bar{b} + d$. This intriguing relation triggered many new ideas (bootstrap, the concept of duality). Backed by the mysterious *linearity* of Regge trajectories relating spins and squared masses of observed hadrons, the duality ideas, via the famous Veneziano amplitude, gave rise to the concept of hadronic strings and to development of string theories in general.

A number of theoretical efforts were devoted to understanding the approximately constant behaviour of the total cross sections at high energies.

To construct a full theory that would include the pomeron trajectory with the maximal 'intercept' that respects the Froissart bound, $\alpha_P(0) = 1$, and would be consistent with unitarity and analyticity proved to be very difficult. This is because multi-pomeron exchanges become essential, which generate branch points in the complex plane of angular momentum ℓ. The simplest branch point of this kind (the two-pomeron cut) was first discovered by Mandelstam in his seminal paper of 1963 [13]. The result was generalized, very elegantly, by V.N. Gribov, I.Ya. Pomeranchuk and K.A. Ter-Martirosian [14]. They showed that Mandelstam's t-channel unitarity analysis could be recast as demonstrating the presence of an ℓ-plane contribution from the four-particle state whose generalization would be the contribution of the N-pomeron cut from the $2N$-particle state.

The t-channel unitarity analysis assumed extensive multi-particle complex angular momentum theory, however. When attempts to develop the needed formalism floundered, Gribov decided that a diagrammatic approach might be more straightforward.

He then developed the general diagram technique known as the Gribov reggeon calculus by considering 'hybrid diagrams' with Regge pole amplitudes connected by the non-planar couplings of Mandelstam. By the end of the 1960s, he had thus formulated the rules for constructing the field theory of interacting pomerons – the Gribov reggeon field theory (RFT). In doing so, he had reduced the problem of high energy scattering to a non-relativistic quantum field theory of interacting particles in 2+1 dimensions. For a long time the Gribov RFT was regarded as 't-channel' in origin. Later, as the structure of the 's-channel' imaginary parts was understood it was realized that the pomeron interaction diagrams directly reflected particle production processes with large rapidity gaps.

With the advent of non-Abelian QFTs, and QCD in particular, Gribov's approaches and calculation techniques were applied in 1976 by his pupils who demonstrated that vector mesons (gluons; intermediate bosons W, Z) *reggeize* in perturbation theory (L. Lipatov; L. Frankfurt and V. Sherman), and so do fermions (quarks; V. Fadin and V. Sherman). The vacuum singularity has also been analysed in perturbative QCD, which analysis resulted in the scattering cross section of two small transverse-size objects *increasing* with energy in a power-like fashion in the restricted energy range (the so-called 'hard' or 'BFKL' pomeron [15]).

A lot of theoretical effort is being invested these days in the programme, formulated by Lipatov, of constructing and solving a (2+1)-dimensional effective QCD pomeron dynamics – a direct offspring of the Gribov RFT.

The last lectures are devoted to the discussion of the problems of the so-called *weak* and *strong* reggeon coupling scenarios.

The problem of high energy behaviour of soft interactions remained unsolved, although some viable options were suggested. In particular, in 'Properties of Pomeranchuk poles, diffraction scattering and asymptotic equality of total cross sections' [16] Gribov showed that a possible consistent solution of the RFT in the weak coupling regime calls for the formal asymptotic equality of *all* total cross sections of strongly interacting particles.

In 1968 V.N. Gribov and A.A. Migdal demonstrated, in a general field theory framework, that in the strong coupling regime the scaling behaviour of the Green functions emerged [17]. Their technique helped to build the quantitative theory of second order phase transitions and to analyse critical indices characterizing the long range fluctuations near the critical point.

In the context of interacting reggeons, the study of the strong coupling regime (pioneered by A.B. Kaidalov and K.A. Ter-Martirosian) led to the introduction of the 'bare' pomeron with $\alpha_P(0) > 1$. The RFT based on t-channel unitarity should enforce the s-channel unitarity as well. The combination of increasing interaction radius and the amplitudes in the impact parameter space which did not fall as $1/\ln s$ (as in the one-pomeron picture) led to logarithmically increasing asymptotic cross sections, resembling the Froissart regime (and respecting the Froissart bound (0.4)).

Nowadays, the popularity of the notion of the 'supercritical' bare pomeron with $\alpha_P(0) > 1$ is based on experiment (increasing total hadron cross sections). Psychologically, it is also supported by the BFKL finding.

Gribov diffusion in the impact parameter space giving rise to energy increase of the interaction radius and to the reggeon exchange amplitude, coexisting fluctuations as a source of branch cuts, duality between hadrons and partons, a common basis for hard and soft elastic, diffractive and inelastic process – these are some of the key features of high energy phenomena in quantum field theories, which are still too hard a nut for QCD to crack.

Added to the main text of the lectures are Gribov's three seminal works produced in the 1970s. They are as follows.

A. The translation of the Gribov lecture at the Leningrad Nuclear Physics Institute Winter School in 1973, in which the understanding of the space–time evolution of high energy hadron–hadron and lepton–hadron processes, in particular the nature of the reggeon exchange from the s-channel point of view, has been achieved. This lecture gives a perfect insight into Gribov's extraordinary way of approaching complicated physical problems of a general nature. He outlined here the general phenomena and typical features that were characteristic for high energy processes in any quantum field theory. The power of Gribov's approach lies in applying the universal picture of fluctuating hadrons to both soft and hard interactions.

B. The paper written in collaboration with V. Abramovsky and O. Kancheli in which the general quantitative relation between the shadowing phenomenon in hadron–hadron scattering, the cross section of diffractive processes and inelastic multi-particle production had been discovered. This is one of the best-known applications of the Gribov RFT known as the 'AGK cutting rules'.

C. Gribov's last work in the subject which was devoted to the intermediate energy range and dealt with interacting hadron fluctuations ('heavy pomeron').

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1
High energy hadron scattering

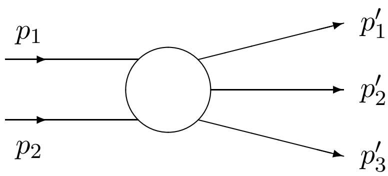
In these lectures the theory of complex angular momenta is presented. It is assumed that readers are familiar with the methods of modern quantum field theory (QFT). Nevertheless we shall briefly recall its basic principles.

1.1 Basic principles

The main experimental fact underlying the theory is the existence of strong interactions between particles of non-zero masses. The theory is constructed for quantities which have a direct physical meaning.

1.1.1 Invariant scattering amplitude and cross section

Such quantities are the scattering amplitudes,



which are supposed to be functions of the kinematical invariants only: $A(p_1, \ldots, p_n) = A(p_i^2, p_i p_k)$. For simplicity, let us begin by considering the scattering of neutral, spinless particles as shown in Fig. 1.1. We use a normalization of the scattering amplitudes such that the kinematical factors arising from the wave functions of the external particles are factorized out. The cross section of any process can be defined in terms of

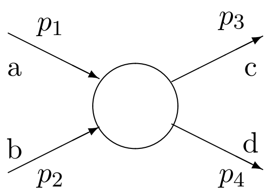


Fig. 1.1. Two-particle scattering

the invariant amplitude A as follows:

$$d\sigma_n = (2\pi)^4 \delta\left(p_1 + p_2 - \sum_i p'_i\right) |A|^2 \prod_{i=1}^{n} \frac{d^3 p'_i}{2p'_{i0}(2\pi)^3} \frac{1}{I},$$
$$I = 4p_{10}p_{20}J = 4\sqrt{(p_1p_2)^2 - m_1^2 m_2^2}\,. \tag{1.1}$$

Here the factor $(2\pi)^4\delta()$ originates from energy–momentum conservation, $d^3p'_i/2p'_{i0}(2\pi)^3$ from the phase space volume; I is the Møller factor which combines the flux density J of the initial particles and $(2p_{10}\,2p_{20})^{-1}$ coming from their wave functions.

1.1.2 Analyticity and causality

It is assumed that the scattering amplitude A is an analytic function of its arguments (for instance it cannot contain terms like $\Theta(p_{i0})$). This assumption is a manifestation of the causality principle. Without analyticity, the scattered waves could appear at their source before being emitted. Additionally, it is natural to conjecture at this point that the growth of the scattering amplitude, as one of the invariants tends to infinity for fixed values of the remaining invariants, is polynomially bounded,

$$|A(p_1, \ldots, p_n)| < (p_ip_j)^N.$$

This assumption is closely related to causality and the locality of the interaction. One needs it in order to write the dispersion representation for the amplitudes (to be able to close the integration contour over an infinitely large circle).

1.1.3 Singularities

It is also assumed that all singularities of the amplitude on the physical sheet have the meaning of reaction thresholds, i.e. they are determined by physical masses of the intermediate state particles. In terms of Feynman diagrams they are the Landau singularities.

1.1.4 Crossing symmetry

We will clarify the meaning of crossing, taking as an example a four-particle amplitude. Since this amplitude depends on the kinematical invariants (and not on the sign of p_{i0}), the same analytic function describes the reaction

$$a(p_1) + b(p_2) \to c(p_3) + d(p_4) \qquad \text{for} \quad p_{10}, p_{20}, p_{30}, p_{40} > 0$$

as well as

$$a(p_1) + \bar{c}(-p_3) \to \bar{b}(-p_2) + d(p_4) \qquad \text{for} \quad p_{10}, p_{40} > 0, \;\; p_{20}, p_{30} < 0$$

and

$$a(p_1) + \bar{d}(-p_4) \to \bar{b}(-p_2) + c(p_3) \qquad \text{for} \quad p_{10}, p_{30} > 0, \;\; p_{20}, p_{40} < 0\,.$$

For an unstable particle, there is the additional reaction $a \to \bar{b} + c + d$ ($p_{10}, p_{30}, p_{40} > 0, p_{20} < 0$).

In fact, the crossing symmetry implies the CPT-theorem – invariance of the amplitude A with respect to the combination of charge conjugation C, space reflection P and time reversal T.

Crossing symmetry follows from the first three assumptions. It can be shown that the same assumptions allow us to prove the spin-statistics relation theorem (the Pauli theorem).

1.1.5 The unitarity condition for the scattering matrix

Unitarity has a simple physical meaning: the sum of probabilities of all processes which are possible at a given energy is equal to unity, $SS^+ = 1$. If $S = 1 + \mathrm{i}\,A$, then

$$\mathrm{i}\,(A - A^+) = -AA^+.$$

Representing the amplitude A as the sum of its real and imaginary parts, $A = \operatorname{Re} A + \mathrm{i} \operatorname{Im} A$, the unitarity condition takes the form

$$2 \operatorname{Im} A = AA^+. \tag{1.2}$$

1.2 Mandelstam variables for two-particle scattering

Let us show how all the above principles work in the case of the four-particle amplitude.