Combinatorics of Symmetric Designs

The aim of this book is to provide a unified exposition of the theory of symmetric designs with emphasis on recent developments. The authors cover the combinatorial aspects of the theory giving particular attention to the construction of symmetric designs and related objects. The last five chapters of the book are devoted to balanced generalized weighing matrices, decomposable symmetric designs, subdesigns of symmetric designs, non-embeddable quasi-residual designs, and Ryser designs. Most results in these chapters have never previously appeared in book form. The book concludes with a comprehensive bibliography of over 400 entries.

Researchers in all areas of combinatorial designs, including coding theory and finite geometries, will find much of interest here. Detailed proofs and a large number of exercises make this book suitable as a text for an advanced course in combinatorial designs.

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To Irina, Tania, and Timur

To Neelima, Aditi, and Sean

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Preface

Design theory is a well-established branch of combinatorial mathematics. The origins of the subject can be traced back to statistics in the pioneering works of R. A. Fisher, F. Yates, and R. C. Bose. From the very beginning, one of the central objects of design theory has been symmetric designs. The prototype of a symmetric design is a finite projective plane, and the theory of symmetric designs borrows its methods and ideas from finite geometries, group theory, number theory, and linear algebra.

It is notoriously difficult to construct an infinite family of symmetric designs or even a single symmetric design. However, in recent years new ideas in constructing symmetric designs have been discovered and new infinite families have been found. The central role in these constructions is played by balanced generalized weighing matrices. These matrices generalize the notion of a symmetric design but until recently they were often regarded as a rather obscure combinatorial object. Now they seem to be a useful tool in unifying different construction methods that have been developed since the 1950s.

This book is primarily a research monograph which aims to give a unifying exposition of the theory of symmetric designs with emphasis on these new developments. The book covers the combinatorial aspects of the theory with particular attention to constructing symmetric designs and related objects. Recent results that have never previously appeared in book format are developed mainly in the last five chapters. These chapters are devoted to balanced generalized weighing matrices, decomposable symmetric designs, subdesigns of symmetric designs, non-embeddable quasi-residual designs, and Ryser designs. The preceding chapters on finite geometries, Hadamard matrices, resolvable designs, t-designs, strongly regular graphs, and difference sets emphasize relations between these objects and symmetric designs.

We believe that this book can also be used as a text for a course in combinatorial designs. We begin with a brief introduction to combinatorial set theory,

including such beautiful results as Fisher's Inequality, the Ray-Chaudhuri–Wilson Inequality, and the Ryser–Woodall Theorem. The proofs of these theorems are elementary, but we hope they may be of interest even to the expert. Both Fisher's Inequality and the Ryser–Woodall Theorem allow us to introduce the notion of a symmetric design even before the formal definition is given in Chapter 2. Chapters 2–4 and 6–9 contain basic material on combinatorial designs, finite geometries, Hadamard matrices, strongly regular graphs, difference sets, and codes. We have included many examples and exercises and presented the proofs of many theorems in a manner suitable for graduate and advanced undergraduate students. Every chapter of the book is concluded by notes containing comments, references, and historical material. We suggest that the following chapters and sections could form a course in combinatorial designs: Chapter 1, Chapter 2 (without Sections 2.7 and 2.8), Chapter 3 (without Section 3.7), and also Sections 4.1, 4.2, 4.3, 6.1, 6.2, 6.3, 6.5, 7.1, 7.2, 9.1, and 9.2. A standard course of linear algebra and the basic notions of combinatorics and abstract algebra should form a sufficient background for this book.

The numbering of theorems, definitions, remarks, and examples is consecutive within each section and includes the chapter and section numbers, so, for instance, Theorem 3.7.10 can be found in Section 3.7. However, equations are numbered consecutively within each chapter. The last two sections of every chapter are **Exercises** and **Notes**. The **Appendix** contains the list of parameters of all known symmetric designs, which are combined into 23 series and 12 sporadic designs. We conclude the book with an extensive **References** section of over 400 entries, all of which are cited in the book.

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