INTRODUCTION TO SPECTROPOLARIMETRY

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Historical introduction

Más has dicho, Sancho, de lo que sabes —dijo don Quijote—; que hay algunos que se cansan en saber y averiguar cosas que, después de sabidas y averiguadas, no importan un ardite al entendimiento ni a la memoria.

—M. de Cervantes Saavedra, 1615.

'You have said more than you realize,' said Don Quijote, 'for there are some, who exhaust themselves in learning and investigating things which, once known and verified, add not one jot to our understanding or our memory.'

Spectropolarimetry, as the name suggests, is the measurement of light that has been analyzed both spectroscopically and polarimetrically. In other words, both the wavelength distribution of energy and the vector properties of electromagnetic radiation are measured with the highest possible resolution and accuracy. Thus, spectropolarimetry embraces a number of techniques used in order to characterize light in the most exhaustive way. Such techniques are ultimately based on a theory that, from its beginnings in the closing years of the nineteenth century, finally grew to maturity in the 1990s. Therefore, under the heading of spectropolarimetry we will find several disciplines, which, despite being interrelated or rather, although our aim is to stress their interrelatedness, may be considered independent.

A historical perspective is always helpful for grasping the importance of physical phenomena and their corresponding explanations. The main objective of this chapter is to give a brief description of the salient events and findings in history related to some of the independent disciplines covered in this book. In particular, polarization phenomena and their treatment, and the Zeeman effect, both on the Sun and in the laboratory, have been especially relevant not only in spectropolarimetry but in the development of natural sciences and are thus deserving of this brief introduction.

As we shall see, polarization phenomena provided the most important observational evidence that finally helped to decide between the corpuscular and undulatory theories of light. In its turn, the Zeeman effect was not only a cornerstone in the development of quantum mechanics but also the key to the discovery and later study of extraterrestrial magnetic fields.

1.1 Early discoveries in polarization

Like many other discoveries in physics and science in general, polarization was brought to the attention of the scientific community quite by chance. A mariner returning to Copenhagen from Iceland brought back several beautiful crystals of what we now know as Iceland spar, or calcite. Some of these crystals, it seems, fell into the hands of Erasmus Bartholin, a Danish physician, mathematician, and physicist, who at that time (1669) was a professor of medicine at the University of Copenhagen. Bartholin observed that images formed through these crystals were double. Moreover, when the crystal was rotated, one image remained in place while the second rotated with the crystal. He rapidly interpreted the phenomenon in terms of rays entering the crystal being immediately split into two, one of which, being stationary during rotation, he termed the “ordinary ray”, and the other – that experiencing the crystal rotation – he called the “extraordinary ray”. As we shall see later, these terms are still in use.

This discovery was later taken up by the Dutch mathematician, astronomer, and physicist Christiaan Huygens, who had no problem in explaining the double refraction with his construction of propagating wavefronts (1690). At any point within the crystal, the light disturbance generated two wavefronts, a spherical one for the ordinary ray and a spheroidal one for the extraordinary ray. Besides this explanation, he contributed the important experimental discovery that the doubly diffracted rays behaved differently from ordinary light when entering a second crystal. Depending on the relative orientation of the two crystals, double refraction may or may not take place again, thus producing (or not) four final rays. He was not able, however, to provide a comprehensive explanation for this new phenomenon. Interestingly, it was Sir Isaac Newton who in 1717 presented the first ideas concerning the reason for double refraction. According to Newton, ordinary light seemed to have the same properties in all directions perpendicular to the direction of propagation, while doubly refracted rays seemed to “have sides”, i.e., to show different properties in different directions in a plane perpendicular to the direction of propagation. Today, we know that this is qualitatively true. However, for Newton these peculiar transversal properties constituted an insuperable objection to accepting the wave theory of light; at that time, accepting the wave theory implied accepting similarities between light and (longitudinal) sound waves.

Most of the scientific debate concerning optics during the eighteenth century was between the rival corpuscular and undulatory theories but double refraction was still a major problem. The undulatory theory started to gain reputation by the turn of the century with the work of the English former physician Thomas Young. Apart from his great discovery of the law of interference in 1801, Young used double refraction as an argument for defending the undulatory theory. On Young's suggestion, in 1802 William Hyde Wollaston studied experimentally the accuracy of the Huygenian construction for the extraordinary ray and found remarkable agreement, although the existence itself of two rays in a single substance was still not completely understood. To get an idea of the importance of the problem suffice it to say that in January 1808 the French Academy proposed "To furnish a mathematical theory of double refraction, and to confirm it by experiment." As the subject for the 1810 physics prize. One of the contestants was Etienne-Louis Malus, an engineering officer in Napoleon's army, who happened to be analyzing solar light reflected by a window through a rhomb of Iceland spar when the two rays showed different intensities! He rightly concluded that the properties hitherto attributed to crystals could also be produced by reflection of light in a variety of substances and he communicated his results to the French Academy by the end of that year, when he coined the term *polarization*. Yet wave-theory defenders were unable to abandon the analogy with sound waves and the debate kept up for some years: including polarization phenomena within that theory was still necessary. The solution to the puzzle took 9 years, during which a number of discoveries occurred. In 1811, Dominique François Jean Arago, a French physicist, discovered optical rotation and in the following year he invented the pile-of-plates polarizer. Also in 1812, the Scottish physicist David Brewster discovered the law named after him concerning polarization by reflection, and the French physicist Jean-Baptiste Biot discovered positive and negative birefringence in uniaxial crystals.

During a visit to Young in 1816, Arago mentioned a new experiment he and Augustin-Jean Fresnel had recently carried out. The result was that two light beams polarized at right angles do not interfere as two rays of ordinary light do, and they always show the same total intensity when reunited, no matter what the path difference for the two beams. This experiment provided Young with the much-sought-after key to the link between the undulatory theory of light and polarization phenomena: the transversality of light vibrations. In 1817, first in a letter to Arago and then in an article for *Encyclopaedia Britannica*, he explained that the assumption of light oscillations perpendicular to the propagation direction fully accounts for the "subdivision of polarised light by reflection in an oblique plane". Young's ideas, communicated by Arago to Fresnel in 1819, were quickly realized by the latter as the main explanation for their experimental results and in fact for all the polarization phenomena known until then. Fresnel rapidly interpreted natural light as a superposition of light polarized in all possible directions and polarization as a manifestation of wave transversality:

So direct light can be considered as the union, or more exactly as the rapid succession, of systems of waves polarised in all directions. According to this way of looking at the matter, the act of polarisation consists not in creating these transverse motions, but in decomposing them into invariable directions, and separating the components from each other; for then, in each of them, the oscillatory motions take place always in the same plane.

These results on polarization laid the foundation for Fresnel's further important discoveries in optics.

1.2 A mathematical formulation of polarization

In his remarkable paper of 1852 entitled "On the composition and resolution of streams of polarized light from different sources", George Gabriel Stokes, Lucasian Professor of Mathematics at the University of Cambridge, established a mathematical formalism ideal for describing the state of polarization of any light beam. Moreover he demonstrated several of the most important properties of polarized light, among which he noted the following:

When any number of independent polarized streams, of given refrangibility, are mixed together, the nature of the mixture is completely determined by the values of four constants, which are certain functions of the intensities of the streams, and of the azimuths and eccentricities of the ellipses by which they are respectively characterized; so that any two groups of polarized streams which furnish the same values for each of these four constants are optically equivalent.

Those four constants referred to by Stokes are what we currently know as Stokes parameters. Unfortunately, the usefulness of the formalism and the importance of the Stokes theorems seems to have been ignored by the scientific community during the following 80 years. In 1929, in a very complete study of the partial polarization of light, the French physicist Paul Soleillet described the Stokes parameters and used them throughout. Interestingly, in the third part of this very paper, a formulation is presented of a theory of anisotropic absorption that is nothing less than the construction of an equation of transfer for polarized radiation. Unfortunately, this paper is still fairly unknown by the astrophysical community. Eighteen years later, in 1947, in his famous series of papers on the radiative equilibrium of stellar atmospheres, and certainly unaware of Soleillet's work, the Indian-born American astrophysicist Subrahmanyan Chandrasekhar published a summary of Stokes's results, emphasizing the importance and usefulness of the formalism which turned out to be particularly well suited to the formulation of a radiative transfer equation in a stellar atmosphere.

One year later, in 1948, Hans Mueller, a professor of physics at the Massachusetts Institute of Technology devised a phenomenological approach to describing the transformation of Stokes parameters by means of 4×4 matrices (nowadays known as Mueller matrices). Since then, Mueller's approach has been extensively used for dealing with partially polarized light. A precursor of this formalism can be found in a paper by Francis Perrin (1942). A few years before Mueller's work, between 1941 and 1947, the American physicist Robert Clark Jones presented his formalism to describe totally polarized light and the transformations between any two totally polarized light beams.

1.3 Discovery of the Zeeman effect

When spectral lines are formed in the presence of a magnetic field, they widen or split into differently polarized components. This phenomenon is known as the Zeeman effect, in honor of its discoverer, the Dutch physicist Pieter Zeeman, who in 1896 found a conspicuous widening of the sodium D lines after switching on an electromagnet. But the origins and precursors of this discovery date back to the middle of the nineteenth century, as does the observation of this particular effect over the surface of the Sun.

In 1845, Michael Faraday discovered that linearly polarized light streaming through a transparent isotropic medium subject to a magnetic field changes the direction of polarization. This relationship between magnetism and light was his inspiration for his final scientific endeavors in 1862 in searching for any trace of the influence of magnetic fields in the spectra of several substances. Unfortunately, he failed to obtain any experimental evidence. Neither did Peter Guthrie Tait of the University of Edinburgh who in 1875, influenced by the mechanical analogies of electromagnetism of William Thomson (Lord Kelvin), had a similar intuition to Faraday's. Ten years later, the Belgian astronomer M. Fievez carried out laboratory experiments in which he did find some indications of a magnetic influence on the sodium spectral lines. He was not able to discriminate magnetic from temperature effects, so he stopped his inquiries at that point.

Unacquainted with this work, Pieter Zeeman, associate professor at the University of Leyden, had the same intuition as Faraday and Tait because of his work on the Kerr effect. His first experiments failed to find any observable effect, and he would have not tried again had he not read by chance, in the *Collected Works* of James Clerk Maxwell, of the final efforts of Faraday. Having had ideas similar to Faraday's encouraged him to take up the experiments with more care. His experimental results were soon forthcoming, and explanations for them came from Hendrik Antoon Lorentz, also a professor at Leyden University. The widening of spectral lines had to be accompanied by a distinct polarization in the wings of the lines. Zeeman and Lorentz shared the Nobel Prize for Physics in 1902. Yet disagreements between experiment and theory were very quickly found, and the Zeeman effect remained unexplained for some 30 years after its discovery. Hence, it constituted an experimental milestone in the development of quantum mechanics. Only after the electron theories of Wolfgang Ernst Pauli (1927; non-relativistic) and Paul Adrien Maurice Dirac (1928; relativistic) could the empirical results be fully understood.

In parallel with laboratory discoveries, astronomical spectroscopy received a great impetus from the middle of the century. In 1866, Sir Joseph Norman Lockyer observed the spectrum of a sunspot. Comparison with the spectrum of the normal solar photosphere revealed a conspicuous widening of the lines. This phenomenon, interpreted nowadays as a result of the Zeeman effect brought about by the sunspot's magnetic field, was observed by many workers until the early 1900s. Unfortunately, nobody realized the relevance of the phenomenon, even well after Zeeman's discovery had been brought to the notice of the astrophysical community.

Motivated by the new morphology of sunspots as seen in H_α photographs, in 1908 George Ellery Hale finally found a convincing explanation for the observed sunspot spectrum: the presence of strong magnetic fields in sunspots. The spectral lines appeared to be widened, split, and in the right state of polarization. This can be thought of as one of the fundamental discoveries of solar physics in the twentieth century. It triggered new and fertile fields of solar and stellar research, for which spectroscopy and polarimetry must be combined in order to exploit to the full the information embedded in the light from heavenly bodies.

1.4 Radiative transfer for polarized light

The specification of the radiation field through an atmosphere that scatters light started as a physical problem in 1871 with the work of the English physicist John William Strutt (Lord Rayleigh) on sky light. Independently of the already-mentioned paper of Soleillet but a few years later, the fundamental equations were formulated and solved by Subrahmanyan Chandrasekhar in his series of papers published in *The Astrophysical Journal*. In the meantime, the works of Arthur Schuster (1905) and Karl Schwarzschild (1906) deserve especial mention because of their revival of the problem mainly within the astrophysical community. By the middle of the twentieth century, however, physicists from other branches became interested in radiative transfer since the same problems seemed to arise in, for instance, the diffusion of neutrons. Most remarkably, the transfer was formulated by Chandrasekhar for polarized radiation since the original problems had to do with light polarized by scattering. Nevertheless, since the 1940s the most extended use of the radiative transfer equation has been in relation to unpolarized problems:

stellar atmospheres have often been assumed isotropic so that just one equation for the total intensity of the light beam is needed.

The study of solar and stellar magnetic fields is an application for which the problem of polarized energy transport is of singular importance. The wealth of information obtained after Hale's discovery of sunspot magnetic fields made it necessary to interpret the spectrum of polarized light observed in the Sun and other stars. Since the mid-1950s, a full theory of polarized radiative transfer has been developed, mostly motivated by the problem of solar/stellar magnetic fields, but whose applications go far beyond astrophysics. Since the theory is relatively young, it would seem appropriate to mention a few landmarks in the literature. The theory builds upon the pioneering work by Wasaburo Unno from Japan (1956) dealing with the formulation – and solution in a simplified Milne–Eddington model – of a radiative transfer equation in the presence of a magnetic field. The work was indeed aimed at describing spectral line formation in the presence of magnetic fields in the solar atmosphere. Only absorption processes were taken into account and the completion of such an equation, including dispersion effects (the so-called magneto-optical effects in the astrophysical literature) was carried out by D. N. Rachkovsky (1962a, 1962b, 1967) from the Ukraine – of course, the solution in that simplified model was also corrected. The formulation, however, was phenomenological and somewhat heuristic and was not put on a firm, rigorous basis until the work by Egidio Landi Degl'Innocenti and Maurizio Landi Degl'Innocenti (1972), who derived the transfer equation for polarized light within the framework of quantum electrodynamics. Later, three derivations of that equation from first principles (basically Maxwell's equations) of classical physics were published: one by John Jefferies, Bruce W. Lites, and Andrew P. Skumanich (1989), another by Jan Olof Stenflo (1991; see also 1994), and a third by Egidio Landi Degl'Innocenti (1992). Many of the developments which follow in this text are based on these three works.

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