

INTRODUCTION TO SPECTROPOLARIMETRY

Spectropolarimetry embraces the most complete and detailed measurement and analysis of light, as well as its interaction with matter. This book provides an introductory overview of the area, which is playing an increasingly important role in modern solar observations. Chapters include a comprehensive description of the polarization state of polychromatic light and its measurement, an overview of astronomical (solar) polarimetry, the radiative transfer equation for polarized light, and the formation of spectral lines in the presence of a magnetic field. Most topics are dealt with within the realm of classical physics, although a small amount of quantum mechanics is introduced where necessary. This text will be valuable for advanced undergraduates, graduates and researchers in astrophysics, solar physics and optics.

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*Instituto de Astrofísica de Andalucía
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To Enriqueta
and our joint project:
Jorge and Ana

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Preface

For the object of the philosopher is not to complicate, but to simplify and analyze, so as to reduce phenomena to laws, which in their turn may be made the stepping-stones for ascending to a general theory which shall embrace them all; and when such a theory has been arrived at, and thoroughly verified, the task of deducing from it the results which ought to be observed under a combination of circumstances which has nothing to recommend it for consideration but its complexity, may well be abandoned for new and more fertile fields of research.

—*G. G. Stokes, 1852.*

Were one asked for a concise description of most astrophysical tasks, one possible answer might be ‘understanding the message of light from heavenly bodies’. Light – or electromagnetic radiation – is the astronomer’s main (almost his sole) source of information. The statement that nobody can measure the physical parameters of the solar atmosphere, although at first sight shocking, merely calls attention to the fact that astrophysics is an observational rather than an experimental science. This characteristic is often forgotten. We do not measure solar or stellar temperatures, velocities, magnetic fields, etc., simply because we do not have thermometers, tachometers, magnetometers, etc., that would permit *in situ* measurements of these parameters. Rather, we are only able to measure light. The astronomical parameters are inferred from these measurements, often with the help of some laboratory physics. Thus, the reliability of such astronomical inferences hinges on the accuracy of measurements of light. It is in this broad sense that spectropolarimetry may be said to embrace all real measurements carried out by astronomers. For, as is clear from the name itself, spectropolarimetry analyzes light as a function of its two most important characteristics: wavelength and state of polarization.

The basic laws or physical tools used in astrophysics are those already known in the laboratory, but sometimes astrophysics offers new insights on how to proceed in the laboratory. A beautiful example can be found in the subject of this book.

Spectropolarimetry was born and mostly developed in the realm of astrophysics – more specifically, in solar physics: the importance of magnetic fields in the overall state of our star has been increasingly more appreciated throughout the course of the twentieth century. Most (if not all) the observable manifestations of solar magnetism have been polarimetric. Great efforts have consequently been made in improving the accuracy of the measurements and in building a theory that permits the analysis of such spectropolarimetric measurements. Both the instrumental and the theoretical developments are of use to a much broader community. Advances in spatial and temporal modulation of the polarimetric signal achieved over the last two decades will doubtlessly be of interest for laboratory polarimetry. On the other hand, the theory of polarized radiation transport not only helps in disclosing the physical parameters of the Sun and other stars, but can be described as a cornerstone in our understanding of the interaction between radiation and matter. It is with this mutual-benefit philosophy in mind that this book has been written: though it is primarily intended for astrophysical applications, most of the concepts and developments described in the book may be of use to other branches of science. The use of astrophysical examples has been minimized as far as possible in order to give a general overview of the topics discussed. Nevertheless, most of the particular examples are still from solar physics.

Being a primordial property of the simplest (and ideal) form of light (the monochromatic, plane, time-harmonic wave), polarization is (fortunately) a valid concept and indeed a characteristic of measurable polychromatic light in the real world. Hence, understanding the message of light necessarily implies (or should imply) a polarimetric analysis of electromagnetic waves in order to exploit fully the information carried by them from heavenly bodies. The other important variable characterizing electromagnetic radiation, frequency, has received wide and intensive attention in many textbooks. Nowadays, the postgraduate student or the young researcher has many authoritative monographs available in which spectroscopy is discussed thoroughly, in both the theoretical and the observational and experimental aspects. This is not the case, however, for polarimetry, whose details are seldom discussed, with very few exceptions. After an introductory historical summary, the first part of the book (Chapters 2–5) is devoted to polarimetry and hence may be of direct application not only to solar physics but to other branches of astrophysics and science in general. But if light and its state of polarization are the primordial observables in astrophysics, the radiative transfer equation (RTE) is crucial for deciphering that encoded information in terms of the physical quantities characterizing the (polarized) light source. Therefore, the second part of the book (Chapters 6–11) deals with the RTE

for polarized light with particular emphasis on the case of magnetized solar atmospheres.[†]

Since the present monograph was conceived as an introductory course for post-graduate or advanced undergraduate students, the reader is assumed to be familiar with such concepts as Maxwell's equations and the Fourier transform. Nevertheless, I have decided to start practically from scratch in order to lay the foundations on which the topics described are built. Therefore, a non-negligible amount of the material presented here has been borrowed from other sources [special mention should be made of the books by Born and Wolf (1993) and Stenflo (1994), and the lecture notes by Landi Degl'Innocenti (1992)].[‡]

Although already pointed out in several works, certain paramount details of polarimetry, such as sign conventions, the distinction between monochromatic and quasi-monochromatic light, the coherent and incoherent superposition of light beams, or the physical meaning of Mueller matrices, need particular attention within the framework of a global description. The kinship between the scalar RTE for unpolarized light and the vector RTE for polarized light (the former being a particular case of the latter) is not emphasized often enough. Likewise, deep-rooted concepts in the community, such as the *height of formation* of spectral lines often hide very important clues to the correct inference of the physical properties of the observational target and should in many cases be forgotten. These are a few reasons that justify the structure of the text.

In Chapter 2, some basic concepts are reviewed such as the description of light as an electromagnetic wave, the monochromatic time-harmonic plane wave, the polarization tensor (or coherency matrix), the Stokes parameters of a monochromatic wave, and the Poincaré sphere. Chapter 3 describes the polarization properties of quasi-monochromatic light. Polychromatic light is introduced as a statistical superposition of monochromatic light. Then, after defining a quasi-monochromatic plane wave, its associated coherency matrix and Stokes parameters are described and their meaning explained from the measurements viewpoint. Finally, the concepts of degree of polarization, natural light, partially and completely polarized light are defined. The transformations suffered by (partially) polarized light after interaction with linear optical systems are dealt with in Chapter 4, where the Mueller matrices and their properties and characterization are presented. A description of the basic block components of a (solar) polarimeter follows, and the chapter ends by outlining the way we measure, i.e., by describing both the spatial and temporal modulation of

[†] I use the term "atmosphere" in the plural because the Sun may be thought of as having many atmospheres, depending on whether we are studying the atmosphere above a granule, an unmagnetized intergranular region, a penumbral filament, an umbra, etc.

[‡] See the references in the recommended bibliography to Chapter 1.

the polarimetric signal. Chapter 5 provides a more in-depth treatment of specific issues germane to *solar* polarimetry, such as environmental (seeing-induced and instrumental) polarization, modulation and demodulation, and a description of current solar polarimeters. Light propagation through low-density, weakly conducting media along with absorption and dispersion phenomena are studied in Chapter 6. The radiative transfer equation for polarized light is discussed in Chapter 7, and the links with the more usual scalar equation for unpolarized light are established. The symmetries and information content of the propagation (“absorption”) matrix and of the source function vector in the local thermodynamic equilibrium (LTE) approximation are also discussed in this chapter. After describing the properties that the Zeeman effect imprints on spectral lines, a specification of the elements of the propagation matrix in the presence of magnetic fields and the formation of lines in a magnetized stellar atmosphere are presented in Chapter 8. The solution of the RTE follows in Chapter 9. The paramount astrophysical problem, namely, that of finding radiative transfer diagnostics that enable the astronomer to interpret the observables (the Stokes profiles) in terms of the physical parameters of the observed atmospheres is dealt with in the final two chapters. Chapter 10 describes evidence for the RTE as the most useful tool in magnetometry. First, the concept of height of formation of spectral lines is discussed critically and a recommendation is made that it be substituted by that of sensitivity of the lines to atmospheric quantities. After linearization of the RTE, the so-called response functions (RFs) are shown to describe such sensitivities fully and their main properties are outlined. Moreover, the sensitivities can be extended to observable parameters derived from the profiles, and through a theoretical generalization of the measurements, generalized RFs can be defined as well. These generalized RFs are the root of the concept of height of formation for measurements. Finally, Chapter 11 summarizes very briefly the aim and the bases of the so-called inversion techniques that are currently the best candidates for inferring the magnetic, dynamic, and thermodynamic properties of solar atmospheres.

Contrary to customary usage, just a few references will be quoted within the text. Instead, a bibliography is recommended at the end of each chapter. The number of cited research papers has been reduced to a minimum but some of these have been compiled purely because of their historical interest. By far most of the auxiliary material the reader may need can be found in books or review papers.

Granada, April 2, 2002

Jose Carlos del Toro Iniesta

Acknowledgements

When I was invited to give a course on ‘Solar polarimetry and magnetic field measurements’ at the Kanzelhöhe Observatory Summer School of 1999, I felt excited at the possibility of summarizing in a didactic manner most of the topics related to my research work of the previous 17 years. Teaching always brings significant rewards to the teacher, and I had been offered a challenging opportunity! I embarked in the project but very soon realized that the material had grown in such a way that to include all my notes in the proceedings would have been unfair to the other lecturers and contributors. That, simply, is the origin of the book you now have in front of you. I must warmly thank Arnold Hanslmeier for his invitation and many of the participants and lecturers for the interest shown and their suggestion that I write a book.

I have been working in Granada, at the Instituto de Astrofísica de Andalucía (CSIC), for the past 3 years, but most of the concepts contained in this book arose from discussions and collaborations with my friends at the Instituto de Astrofísica de Canarias. Names that come to mind are Manolo Vázquez, Fernando Moreno-Insertis, Jorge Sánchez Almeida, Javier Trujillo Bueno, Inés Rodríguez Hidalgo, Carlos Westendorp Plaza, Luis Bellot Rubio, and, most importantly, Manolo Collados, Valentín Martínez Pillet, and Basilio Ruiz Cobo. I hope I have not forgotten anybody. From all of these colleagues I have benefited in terms not only of science but also of friendship. Among the colleagues and friends from abroad, I would like to thank Bruce Lites, Andy Skumanich, Arturo López Ariste, and Héctor Socas-Navarro from the High Altitude Observatory (Boulder, Colorado), and, of course, Meir Semel from the Observatoire de Paris-Meudon and Egidio Landi Degl’Innocenti from the Università di Firenze. I learnt many things from them. Luis, Arturo, Manolo, Héctor and Basilio critically reviewed individual chapters, and Antonio Claret, of the IAA, reviewed the whole text. I am deeply indebted to them all, but, of course, any errors that may still remain are entirely my responsibility.

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J.C.T.I.