Codes, Cryptology and Curves with Computer Algebra

This well-balanced text touches on theoretical and applied aspects of protecting digital data. The reader is provided with the basic theory and is then shown deeper fascinating detail, including the current state of the art. Readers will soon become familiar with methods of protecting digital data while it is transmitted, as well as while the data is being stored.

Both basic and advanced error-correcting codes are introduced together with numerous results on their parameters and properties. The authors explain how to apply these codes to symmetric and public key cryptosystems and secret sharing. Interesting approaches based on polynomial systems solving are applied to cryptography and decoding codes. Computer algebra systems are also used to provide an understanding of how objects introduced in the book are constructed, and how their properties can be examined. This book is designed for Masters-level students studying mathematics, computer science, electrical engineering or physics.

RUUD PELLIKAAN has tenure at the Technische Universiteit Eindhoven, The Netherlands where his research has shifted from a devotion to coding theory, particularly algebraic geometry codes and their decoding, to code-based cryptography. He previously served as an associate editor of the *IEEE Transactions of Information Theory* and has organised several conferences.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521817110
DOI: 10.1017/9780511982170

First published 2018

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library.

ISBN 978-0-521-81711-0 Hardback
ISBN 978-0-521-52036-2 Paperback

If three men be walking together,
and (only) one of them be under a delusion,
they may yet reach their goal, the deluded being the fewer;
but if two of them be under the delusion, they will not do so,
the deluded being the majority.

"Heaven and Earth" chapter 14
Zhuangzi (370–287 BC) [364]

Contents

Preface

An early version of this book was a handwritten manuscript from around 1990. In June 2001 a synopsis was written by invitation of Cambridge University Press with the working title "The construction and decoding of algebraic geometry codes," or "Algebraic geometry and its applications (in error-correcting codes and cryptography)." That proposal was accepted, but with no indication of a deadline. So originally its aim was a book on algebraic geometry codes. As time passed more and more co-authors joined the team: Xin-Wen Wu in 2004, Stanislav Bulygin in 2007 and finally Relinde Jurrius in 2012.

Early versions of chapters were written on algebraic geometry codes, elementary coding theory, list decoding Reed–Muller codes, decoding algorithms and Gröbner bases, cryptography, weight enumerators and its generalizations and relations with matroid theory that appeared in books and journals [68, 69, 70, 71, 177, 184, 186, 266].

The prerequisites of this book are: elementary logical reasoning and naive set theory, some combinatorics and probability theory. Furthermore: linear algebra, the beginnings of group theory, the algebra of rings and fields. We will go into the details of polynomial rings and finite fields in several chapters.

The first six chapters of the book on the construction, properties and decoding of error-correcting codes are self-contained. This can be used as a course in Coding Theory of four hours a week during a semester in the first year of the Masters. It is advised to use Chapter 12 from the start to practise the theory with computer algebra systems.

The second half of the book on complexity theory, cryptology, Gröbner bases applied to codes and cryptosystems and algebraic geometry codes is more advanced. This can be used for a course in the second year of a Masters degree or can be read individually as a Capita Selecta. It is

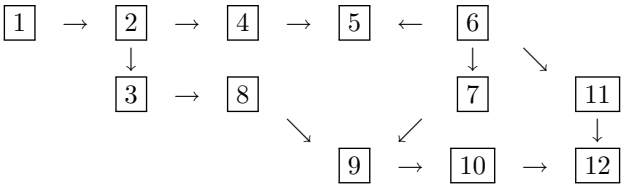
also a good starting point for a project or assignment. In the Notes at the end of every chapter we give ample references to further reading.

We thank the editors of Cambridge University Press: David Tranah, Jonathan Walthoe, Roger Astley, Clare Dennison and Abigail Walkington for their advice and patience.

The logical dependency between the chapters

1 Error-correcting codes
2 Code constructions and bounds
3 Weight enumeration
4 Cyclic codes
5 Polynomial codes
6 Algebraic decoding
7 Complexity and decoding
8 Codes and related structures
9 Cryptology
10 Gröbner bases for coding and cryptology
11 Codes on curves
12 Coding and cryptology with computer algebra

is given in the following diagram:



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bulygin5@googlemail.com, Stanislav Bulygin made the lion's share of his contribution while at Technische Universität Kaiserslautern and Technische Universität Darmstadt (both Germany) in 2008–2013

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