Designing Inclusion

Tools to Raise Low-end Pay and Employment in Private Enterprise

edited by

Edmund S. Phelps

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge, CB2 2RU, UK
40 West 20th Street, New York, NY 10011–4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2003

Printed in the United Kingdom at the University Press, Cambridge

Typeface Plantin 10/12 pt. *System* LaTeX 2_ε [TB]

A catalogue record for this book is available from the British Library

ISBN 0 521 81695 5 hardback

Contents

List of figures *page* vii
List of tables ix
List of contributors xi

Introduction 1
EDMUND S. PHELPS

1 Low-wage employment subsidies in a labor-turnover model of the "natural rate" 16
HIAN TECK HOON AND EDMUND S. PHELPS

2 Taxes, subsidies and equilibrium labor market outcomes 44
DALE MORTENSEN AND CHRISTOPHER PISSARIDES

3 Learning-by-doing versus on-the-job training: using variation induced by the EITC to distinguish between models of skill formation 74
JAMES J. HECKMAN, LANCE LOCHNER AND RICARDO COSSA

4 Unemployment vouchers versus low-wage subsidies 131
J. MICHAEL ORSZAG AND DENNIS J. SNOWER

Index 161

Figures

1.1	Wealth supply and asset demand	*page* 23
1.2	Labor, product and capital market equilibrium	27
1.3	Effects of a flat subsidy	30
1.4	Comparison of flat and graduated subsidies	32
1.5	Effects of a flat subsidy on hiring and employment	38
2.1	Equilibrium market tightness and reservation productivity	56
3.1	Earned Income Tax Credit	75
3.2	Simulated effects of EITC on investment (OJT model)	107
3.3	Simulated effects of EITC on hours worked (OJT model)	107
3.4	Simulated effects of EITC on human capital (OJT model)	108
3.5	Simulated effects of EITC on wage rates (OJT model)	108
3.6	Simulated effects of EITC on wage income (OJT model)	109
3.7	Simulated effects of EITC on hours worked (LBD model)	115
3.8	Simulated effects of EITC on human capital (LBD model)	115
3.9	Changes in hourly wages owing to increases in hours worked (LBD model)	117
3.10	Simulated effects of EITC on wage income (LBD model)	117
4.1	The structure of the model	137
4.2	The effect of changes in wages and employer firing propensity (ϕ) on the reward to work	141
4.3	The effect of changes in wages and employer firing propensity (ϕ) on unemployment	141
4.4	The effect of changes in the discount factor on the reward to work	142
4.5	The effect of changes in risk aversion (γ) on the reward to work	142
4.6	The value to dead-end workers of being employed with and without an unemployment voucher	144
4.7	The value to dead-end workers of being employed with and without a wage subsidy	144
4.8	The value to dead-end workers of being unemployed with and without an unemployment voucher	145

4.9 The value to dead-end workers of unemployment at different durations with a low-wage subsidy 145
4.10 Change in employment at different durations with an unemployment voucher 146
4.11 Change in unemployment at different durations with an unemployment voucher 147
4.12 Change in unemployment at different durations with a low-wage subsidy 147
4.13 Change in employment at different durations with a low-wage subsidy 148
4.14 The value to upwardly mobile workers of being employed with and without an unemployment voucher 149
4.15 The value to upwardly mobile workers of being unemployed with and without an unemployment voucher 150
4.16 Optimal duration-dependent balanced-budget payroll policy with flat wages 151
4.17 Optimal duration-dependent balanced-budget payroll policy with increasing wages 151

Tables

2.1 Baseline parameter values *page* 60
2.2 Outcomes: high skill ($p = 1.4$) 62
2.3 Outcomes: low skill ($p = 0.75$) 62
2.4 Effects of an employment subsidy 65
2.5 Effects of a hiring subsidy 65
2.6 Effects of a "low-skill" subsidy and "high-skill" surtax 68
3.1 Earned Income Tax Credit for 1994 90
3.2 Distribution of households qualifying for the EITC 91
3.3 Distribution of earnings for working female high-school dropouts over the EITC schedule (household heads with children) 92
3.4 Distribution of earnings for working female high-school graduates over the EITC schedule (household heads with children) 93
3.5 Estimated lifecycle progression through the EITC schedule (OJT simulations based on 1994 EITC schedule for two children) 106
3.6 Effects of EITC (OJT model) 109
3.7 Annual compensation necessary to make people indifferent between employment and non-employment (OJT simulations based on 1994 EITC schedule for two children) 110
3.8 Employment rates and average human capital levels (OJT simulations based on 1994 EITC schedule for two children) 111
3.9 Effects of EITC on potential skill levels (OJT simulations based on 1994 EITC schedule for two children) 112
3.10 Effects of EITC on skills supplied to the market (OJT simulations based on 1994 EITC schedule for two children) 114
3.11 Estimated lifecycle progression through the EITC schedule (LBD simulations based on 1994 EITC schedule for two children) 116
3.12 Effects of EITC when imposed (LBD model) 118

3.13 Annual compensation necessary to make people indifferent between employment and non-employment (LBD simulations based on 1994 EITC schedule for two children) 118
3.14 Employment rates and average human capital levels (LBD simulations based on 1994 EITC schedule for two children) 119
3.15 Effects of EITC on potential skill levels (LBD simulations based on 1994 EITC schedule for two children) 120
3.16 Effects of EITC on skills supplied to the market (LBD simulations based on 1994 EITC schedule for two children) 121
3A.1 Definition of EITC regions for CPS calculations 126
3D.1 Parameter values for the OJT model 128
3D.2 Parameter values for the LBD model 128
3D.3 Goodness of fit (OJT model): weighted sum of squared differences 129
3D.4 Goodness of fit (LBD model): weighted sum of squared differences 129

1 Low-wage employment subsidies in a labor-turnover model of the "natural rate"

Hian Teck Hoon and Edmund S. Phelps

Abstract

This paper models two kinds of wage subsidy in a model of the natural rate having a continuum of workers ranked by their productivity – a flat wage subsidy and a graduated wage subsidy, each financed by a proportional payroll tax. In the small open economy case, with the graduation as specified, we show that both schemes expand employment throughout the distribution; for those whose productivity is sufficiently far below the mean, take-home pay is unambiguously up, though the tax financing lowers take-home pay at the mean and above. For any particular class of workers paid the same amount of the wage subsidy under the two plans, the graduated plan expands employment more. In the closed economy case, employment is increased for workers whose productivity levels are below or equal to the mean but the interest rate is pulled up, and that may cause employment to fall at productivity levels sufficiently far above the mean.

There is considerable agreement that the extraordinarily low commercial productivity of active-age persons in the lower reaches of the distribution relative to median productivity is the number one social problem of our time. In creating a huge wage gap it makes the less productive incapable of supporting a family, or in some cases themselves (in a way that meets community standards of decency at any rate), and having access to mainstream community life. In reducing the wage incentives that private enterprise can afford to offer low-wage workers relative to their other resources and attractions, it worsens unemployment and non-participation. Both sets of effects operate in turn, especially in areas where there is a high concentration of these effects, to increase dependency on welfare and property crime, spread drug use and violence, widen illegitimacy and blight the upbringing of children (Murray, 1984; Phelps, 1994b, 1997; Freeman, 1996; Wilson, 1996).

There is far less agreement on what, if anything, would be useful to do about it. An important line of thinking, however, looks to wage subsidies

This chapter is a revised version of a paper presented at a conference on low-wage employment subsidies sponsored by the Russell Sage Foundation, New York, December 1997.

of one kind or another. The pioneers were Cecil Pigou (1933) and Nicholas Kaldor (1936), who studied the conditions for employment subsidies to be self-financing. Targeted hiring subsidies were championed by Daniel Hamermesh (1978), Michael Hurd and John Pencavel (1981) and Robert Haveman and John Palmer (1982). The employment-expanding effects of a constant employment subsidy were studied by Richard Jackman and Richard Layard (1986). Phelps argued informally for a graduated employment subsidy to raise low-end wage rates (Phelps, 1994a) and to reduce unemployment (Phelps, 1994b) as a counterweight to the welfare system. A hiring subsidy targeted at the long-term unemployed has been championed by Dennis Snower (1994). Wage subsidies were urged to counter the effects of payroll taxes by Jacques Dréze and Edmond Malinvaud (1994). Christopher Pissarides (1996) has studied the effects of such tax relief.

These analyses focus on the subsidies' near-term effects. None of the papers expressly argues that there would be a permanent effect on unemployment. Some of the authors may have thought the effect was only temporary but a way to buy valuable time. To study the long-term effects, however, requires an intertemporal model in which workers accumulate wealth and firms invest in capital of one or more kinds according to expectations of the future and interest rates.

As a comparative exercise, the first section undertakes a neoclassical analysis of the effects in the steady state of a flat (constant) subsidy, financed by a proportional payroll tax on the equilibrium level of manhours supplied. We show that wealth decumulation serves ultimately to eliminate the employment decline first brought by the tax, and wealth accumulation operates to eliminate all the employment gains brought by the subsidy. The employment effect is ultimately neutralized, although the take-home wage is increased for low-wage workers.

We then shift to the theory of the natural rate of unemployment. Using our labor-turnover model, with its incentive wage, we study two employment subsidies: a flat (constant) subsidy and a graduated subsidy that decreases with the wage rate and vanishes asymptotically at the top – each program financed by a flat-rate payroll tax (as if resulting external benefits brought no revenue gains). In this model (Phelps, 1968, 1994c; Hoon and Phelps, 1992), quitting by employees poses an incentive problem for the firm, since it must invest in the firm-specific training of workers to make them functioning employees and such an investment is lost whenever an employee quits. The problem prompts firms to drive up the going wage. This leads in turn to involuntary unemployment in labor-market equilibrium. Our 1992 paper posited worker-savers in overlapping cohorts to obtain a general equilibrium framework with which to endogenize the rate of interest or the accumulation of net foreign assets. This chapter introduces a continuum of workers differentiated by productivity in each cohort.

The gist of our findings can be indicated. Owing to incentive-wage considerations, the two schemes permanently expand employment in the long run. The proportional payroll tax used to finance the subsidy is neutral for employment. With employment unchanged, the payroll tax lowers take-home pay in the same proportion for every type of worker but non-wage income is also reduced by the same proportion. As a result, the incentive-wage condition is invariant to the proportional payroll tax in the long run. The subsidy, however, is non-neutral. If, before, a penny increase in hourly labor compensation by the firm had a marginal benefit equal to marginal cost at the original employment rate, it must now have a marginal benefit less than the marginal cost because, with take-home pay up, since it drives a wedge between take-home pay and hourly labor compensation (net of subsidy) that additional penny now has a smaller impact on quitting. Hence firms cut their hourly compensation, and as a result employment is expanded throughout the distribution in the long run.

For low-wage workers, there is an added boost to employment in the short run. Given net wealth and the interest rate, the higher take-home pay induces a decline in the propensity to quit. The result is a rightward shift of the zero-profit curve and an additional rightward shift of the incentive-wage curve on top of the wedge caused by the subsidy. In the long run, wealth accumulation leads to a proportionate rise of non-wage income at given employment, thus shifting the zero-profit curve back to its original position. The incentive-wage curve also shifts leftward but not by enough to eliminate the added boost. The net result, then, for low-wage workers is that the expansionary effect on employment is even larger in the short run than in the long run.

The long-run question in the closed economy case is the subsidies' effect on the rate of interest and the effect in turn on wages and employment. Here we find that, if the zero-profit curve is elastic, aggregate wealth supply is increased, but it increases by less than the increase in asset demand. The result is a rise in the rate of interest. However, for workers whose productivity levels are below or equal to the mean, employment is expanded; at productivities far enough below the mean, take-home pay is also increased.

We also found that the graduated scheme, besides having (for the same subsidy rate at the bottom) a lighter budgetary burden than the constant subsidy, has an extra downward impact on hourly labor cost, as firms moderate wage rates above the bottom to win a larger subsidy, with the result that employment receives an extra boost. Such an effect raises the fear that some middle-wage workers would see their wage reduced on balance. We show, however, that unless the subsidy tapers off too fast no such wage effect occurs. Finally, we show that the gross hiring rate is increased the most for low-wage workers.

The paper is organized as follows. Section 1 analyzes the effects of a flat (constant) subsidy in a neoclassical model. Section 2 presents the basic features of the labor-turnover model with a continuum of workers exhibiting constant marginal training cost. Section 3 studies the incidence of the subsidies in the steady-state, general equilibrium model of the small open economy, and section 4 analyzes the closed economy case. Section 5 briefly discusses the case of rising marginal training cost in the small open economy case. Section 6 concludes.

1 Neoclassical theory

We follow the treatment by Olivier Blanchard (1985) of finitely lived agents with no bequest in a one-sector setup (see Kanaginis and Phelps, 1994, and Phelps, 1994c: ch. 16.) In each cohort, the workers form a continuum when ranked by their respective potential productivity levels. The productivity, or ability, of worker input at location i in this continuum is measured by a labor-augmenting, hence Harrod-neutral, parameter denoted Λ_i. There is a known and unvarying distribution of Λ_i in the working population, which we normalize to one. The proportion of workers with productivity level Λ_i or less is $G(\Lambda_i)$ and the density function is $g(\Lambda_i) = G'(\Lambda_i)$. We call a worker with productivity level Λ_i a type-i worker.

Each agent of type i derives utility from consumption and leisure, which we assume are additively separable and take the log form. He or she has a finite life and faces an instantaneous probability of death θ that is constant throughout life. Solving the agent's problem, and denoting aggregate variables by capital letters, we obtain $C_i = (\theta + \rho)[H_i + W_i]$ and $(\bar{L} - L_i)/C_i = 1/v_i^{\mathrm{h}}$, where C_i is consumption, L_i is labor supply, H_i is human wealth, and W_i is nonhuman wealth per member of the type-i workforce. Here ρ is the time preference parameter, $\bar{L}$ is total time available, and v_i^{h} is the real hourly household wage received by a type-i worker, which is related to the hourly labor cost to the firm of a type-i worker, v_i^{f}, by $v_i^{\mathrm{f}} \equiv (1 + \tau)v_i^{\mathrm{h}} - s_i$, τ being the proportional payroll tax rate. Under the flat subsidy scheme, s_i equals s^{F}, a constant. Under a graduated subsidy scheme, s_i is a decreasing function of the wage paid by the firm to each type-i worker, denoted $s_i = S(v_i^{\mathrm{f}})$, and tapers off asymptotically. We impose throughout the conditions $S'(v_i^{\mathrm{f}}) < 0$ and $|S'(v_i^{\mathrm{f}})| < 1$.

In the small open economy, the path of the domestic interest rate conforms to the exogenously given world interest rate, r^*: $r = r^*$, r^*

a constant > 0. The level of net external assets adjusts endogenously to bring about this condition. The steady-state H_i equals $v_i^{\mathrm{h}} L_i/(r^* + \theta)$ and non-wage income of a type-i worker is given by $y_i^{\mathrm{w}} \equiv (r^* + \theta) W_i$, θW_i being actuarial dividend. In the steady state, setting $\dot{C}_i = 0$, we also have $r^* = \rho + [\theta(\theta + \rho) W_i/C_i]$. This can be rewritten, after some substitutions, as

$$r^* = \rho + \frac{\theta}{1 + \left(\frac{v_i^{\mathrm{h}} \bar{L}}{y_i^{\mathrm{w}}}\right)\left(\frac{L_i}{\bar{L}}\right)}. \tag{1.1}$$

The steady-state labor supply relation in manhours can also be expressed as

$$\frac{L_i}{\bar{L}} = \frac{1 - \left[\frac{\theta+\rho}{r^*+\theta}\right]\left(\frac{v_i^{\mathrm{h}} \bar{L}}{y_i^{\mathrm{w}}}\right)^{-1}}{1 + \left[\frac{\theta+\rho}{r^*+\theta}\right]}. \tag{1.2}$$

Turning to the production side, let the production function be written as $Y = [\int_{\underline{\Lambda}}^{\infty} \Lambda_i L_i g(\Lambda_i) d\Lambda_i] f(K/\int_{\underline{\Lambda}}^{\infty} \Lambda_i L_i g(\Lambda_i) d\Lambda_i)$, where $\underline{\Lambda}$ is the minimum productivity level and K is capital stock. Firms' optimal choice of labor and the capital–labor ratio, $k \equiv (K/\int_{\underline{\Lambda}}^{\infty} \Lambda_i L_i g(\Lambda_i) d\Lambda_i)$, imply

$$\frac{v_i^{\mathrm{f}}}{\Lambda_i} = f(k) - kf'(k); \tag{1.3}$$

$$r^* = f'(k). \tag{1.4}$$

The given world interest rate, r^*, pins down the optimal capital–labor ratio, k. Consequently, the wage paid by the firm, v_i^{f}, is pinned down, being directly proportional to Λ_i. Observe that the wage-to-non-wage income ratio in (1.1) is an implicit function of r^* at each L_i:

$$\frac{v_i^{\mathrm{h}} \bar{L}}{y_i^{\mathrm{w}}} = \Upsilon\left(r^* - \rho, \frac{L_i}{\bar{L}}\right); \ \Upsilon_1 < 0, \ \Upsilon_2 < 0. \tag{1.5}$$

Using this in (1.2), we obtain a reduced-form labor supply relation in the steady state:

$$\frac{L_i}{\bar{L}} = \frac{1 - \left[\frac{\theta+\rho}{r^*+\theta}\right]\left[\Upsilon\left(r^* - \rho, \frac{L_i}{\bar{L}}\right)\right]^{-1}}{1 + \left[\frac{\theta+\rho}{r^*+\theta}\right]}. \tag{1.6}$$

This equation uniquely determines the labor supply in manhours and is independent of the tax and subsidy rates. It is also independent of Λ_i.

To understand this result, we notice that the labor demand curve in the $(L_i/\bar{L}, v_i^{\mathrm{f}})$ plane is infinitely elastic. With wealth and hence y_i^{w} given, the labor supply schedule is upward sloping. Under a balanced-budget policy, the flat subsidy case yields a convenient expression for the tax rate, namely, $\tau = s^{\mathrm{F}}/v^{\mathrm{h}}_{\mathrm{mean}}$, where $v^{\mathrm{h}}_{\mathrm{mean}} \equiv \int_{\underline{\Lambda}}^{\infty} v_i^h g(\Lambda_i) d\Lambda_i$. For an employee whose $\Lambda_i < \Lambda_{\mathrm{mean}}$, the tax liability ($\tau v_i^{\mathrm{h}}$) is therefore less than the subsidy (s^{F}). Hence, at given y_i^{w}, low-wage workers increase their equilibrium labor supply. Wealth accumulation then brings their y_i^{w} up until the original $v_i^{\mathrm{h}}/y_i^{\mathrm{w}}$ is restored. On the other hand, for employees whose $\Lambda_i > \Lambda_{\mathrm{mean}}$, their v_i^{h} is reduced. Such high-wage workers decumulate wealth until once again the original $v_i^{\mathrm{h}}/y_i^{\mathrm{w}}$ is restored. Thus, in the long run, the tax-subsidy scheme is neutral for employment for all workers throughout the distribution. A similar argument holds for the graduated subsidy scheme.

In the closed economy case, the essential task is to endogenize the rate of interest. One approach to the problem is to work toward a diagram involving an asset demand curve and a wealth supply schedule, the intersection giving us the general equilibrium rate of interest. Using the following two conditions:

$$r = \rho + \frac{\theta}{1 + \left(\frac{v_i^{\mathrm{h}} \bar{L}}{y_i^{\mathrm{w}}}\right)}, \tag{1.7}$$

$$\frac{L_i}{\bar{L}} = \frac{1 - \left[\frac{\theta+\rho}{r+\theta}\right]\left(\frac{v_i^{\mathrm{h}} \bar{L}}{y_i^{\mathrm{w}}}\right)^{-1}}{1 + \left[\frac{\theta+\rho}{r+\theta}\right]}, \tag{1.8}$$

we prove in the appendix that we can write $L_i/\bar{L}$ as a decreasing function of r, given ρ and θ, that is,

$$\frac{L_i}{\bar{L}} = \psi(r; \rho, \theta); \ \psi'(r) < 0. \tag{1.9}$$

Using the firm's optimal condition $r = f'(k)$ and (1.9), the aggregate asset demand given by

$$A = k \int_{\underline{\Lambda}}^{\infty} \Lambda_i \bar{L} \psi(r) g(\Lambda_i) d\Lambda_i \tag{1.10}$$

is decreasing in r.

The average supply of wealth per member of the type-i workforce is obtained by substituting $y_i^{\mathrm{w}} \equiv (r+\theta) W_i$ in (1.8):

$$W_i = \left[\frac{\left(v_i^{\mathrm{h}} \bar{L}\right)\left(\frac{L_i}{\bar{L}}\right)}{r+\theta}\right]\left[\frac{r-\rho}{\theta+\rho-r}\right]. \tag{1.11}$$

Excluding the case where $r - \rho > \theta$, we have a well-defined steady state with the righthand side of (1.11) being unambiguously positive. Observe that the first bracketed term in (1.11) is simply human wealth per type-i

worker and, for a given after-tax real wage ($v_i^{\mathrm{h}} L_i$), human wealth, H_i, is decreasing in r. On this account, W_i falls as r rises. On the other hand, a rise of r has a positive effect on W_i on account of the second bracketed term, W_i/H_i. The total supply of wealth per worker is given by $W \equiv \int_{\underline{\Lambda}}^{\infty} W_i g(\Lambda_i) d\Lambda_i$. Using (1.11), we obtain

$$W = \left[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}\right] \int_{\underline{\Lambda}}^{\infty} v_i^{\mathrm{h}} L_i g(\Lambda_i) d\Lambda_i.$$

Under a balanced budget, we get

$$W = \left[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}\right] \int_{\underline{\Lambda}}^{\infty} v_i^{\mathrm{f}} L_i g(\Lambda_i) d\Lambda_i. \tag{1.12}$$

Using $(v^{\mathrm{f}}/\Lambda_i) = f(k) - kf'(k)$ and (1.9), and noting that k is a decreasing function of r, we obtain an expression of total wealth supply as a function of the rate of interest:

$$W = \left[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}\right] \int_{\underline{\Lambda}}^{\infty} [f(k) - kf'(k)]\psi(r)\bar{L}\Lambda_i g(\Lambda_i) d\Lambda_i. \tag{1.13}$$

What is the shape of the supply of wealth? There are two opposing forces. In the general equilibrium, an increase of r lowers the real wage as well as the supply of manhours; and, as remarked above, it lowers the present value of these expected earnings. So human wealth is reduced. However, the second bracketed term in (1.11) works to increase the desired supply of wealth as r rises. At r sufficiently low that W_i is at or near zero, the former effects are outweighed by the latter, though at sufficiently high r the opposite may occur. Hence the per worker supply of wealth schedule is upward sloping initially but at very high r may bend backward. In the same plane, per worker demand for the domestic assets is downward sloping. We will suppose that the equilibrium r is unique or that only the lowest equilibrium r is empirically relevant (see figure 1.1). The important thing to observe from (1.10) and (1.13) is that the pair of equations are independent of the tax-subsidy parameters. Hence the balanced-budget tax-subsidy policy is neutral for the rate of interest and, consequently, also neutral for employment.[1] Nevertheless, for low-wage workers, their take-home pay is increased.

[1] Another way to see that the policy is neutral for the rate of interest is to use the requirement that aggregate supply be equal to aggregate demand. Equating the aggregate demand to aggregate supply in the equation, $(r-\rho)\int_{\underline{\Lambda}}^{\infty} C_i g(\Lambda_i) d\Lambda_i = \theta(\theta+\rho)\int_{\underline{\Lambda}}^{\infty} W_i g(\Lambda_i) d\Lambda_i$, we obtain, $r = \rho + [\theta(\theta+\rho)k/f(k)]$, which, noting that k is decreasing in r, determines the general equilibrium r independently of the tax-subsidy parameters.

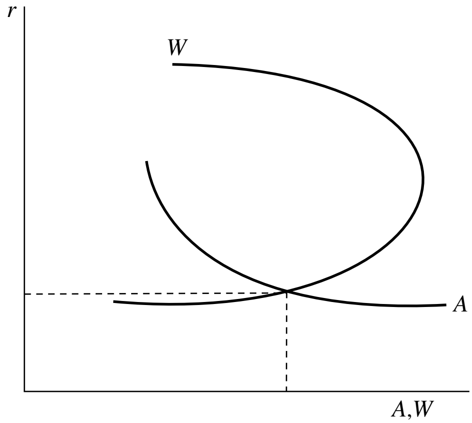

Figure 1.1 *Wealth supply and asset demand*

2 Basic features of the economy in modern equilibrium theory

The preceding neoclassical theory has difficulty explaining why, under plausible assumptions, the policy shift and other aggregate shocks experienced in recent decades should cause large changes in equilibrium labor input and national income. That is in part because the theory does not allow for *unemployment*; rather, changes in labor input are attributable entirely to variations in the work week.

To study the effects of the tax-subsidy schemes on the equilibrium path of unemployment, we need to draw on modern equilibrium theory, which sees unemployment as structural in nature and traces its vicissitudes to changes in the structure of the economy (Phelps, 1994c). At the center of this theory is the relationship between the firm and the employee arising from their incentives in the modern setting of asymmetric information. The economics of incentive (or efficiency) wages plays a key role in generating involuntary unemployment and shaping its equilibrium path.

There are many identical firms. For convenience we may think of them as fixed in number (normalized to one) and equal in size. Consider the representative firm j. Its problem is to choose the wage and hiring–training policies that maximize

$$\int_0^\infty \int_{\underline{\Lambda}}^\infty N_{jit}\{\Lambda_i[1-\beta h_{jit}] - v^{\mathrm{f}}_{jit}\} g(\Lambda_i) e^{-\int_0^t r_v dv} d\Lambda_i dt,$$

which is the present value of the stream of real quasi-rents, subject to

$$\dot{N}_{jit} = N_{jit}\left[h_{jit} - \zeta\left(\frac{z^{\mathrm{he}}_{it}}{v^{\mathrm{h}}_{jit}}, \frac{y^{\mathrm{w}}_{it}}{v^{\mathrm{h}}_{jit}}\right) - \theta\right]$$

and given N_{ji0}. Note that s_i is implicit in v^{h}_{ji} and v^{f}_{ji}, given τ. (Since to simplify we will initially work with constant marginal training cost, we also assume that h_{jit} is bounded, $0 \leq h_{jit} \leq \bar{h}$.) Here, N_{jit} is the stock of type-i employees at the representative firm j taken as a ratio to the type-i workforce (equivalently, the rate of employment among type-i workers), βh_{jit} is the fraction of their working time type-i employees devote to training new hires, h_{jit} is the gross hiring rate of new type-i recruits, ζ similarly measured is the quit rate, and z^{he}_{it} is a proxy for the expected value of real wage earnings of a type-i worker employed at firm j if he or she quits.[2]

We may write the current-value Hamiltonian as

$$\int_{\underline{\Lambda}}^{\infty} \{\Lambda_i[1-\beta h_{jit}] - v^{\mathrm{f}}_{jit} \\ + q_{jit}\left[h_{jit} - \zeta\left(z^{\mathrm{he}}_{it}/v^{\mathrm{h}}_{jit}, y^{\mathrm{w}}_{it}/v^{\mathrm{h}}_{jit}\right) - \theta\right]\} N_{jit} g(\Lambda_i) d\Lambda_i,$$

where q_{jit} is the co-state variable.[3] It measures the shadow value of a type-i worker after training by the employer. First-order necessary conditions (which are also sufficient under our assumptions) are given by

$$\left.\begin{array}{ll} h_{jit} = \bar{h} & \text{if } q_{jit} > \Lambda_i\beta; \\ h_{jit} = 0 & \text{if } q_{jit} < \Lambda_i\beta; \\ h_{jit} \in [0, \bar{h}] & \text{if } q_{jit} = \Lambda_i\beta; \end{array}\right\} \tag{1.14}$$

$$N_{jit}\left\{-1 + q_{jit}\left[\left(\frac{z^{\mathrm{he}}_{it}}{v^{\mathrm{h2}}_{jit}}\right)\zeta_1 + \left(\frac{y^{\mathrm{w}}_{it}}{v^{\mathrm{h2}}_{jit}}\right)\zeta_2\right]\frac{dv^{\mathrm{h}}_{jit}}{dv^{\mathrm{f}}_{jit}}\right\} = 0; \tag{1.15}$$

$$\dot{q}_{jit} - r_t q_{jit} = -\left\{\Lambda_i - v^{\mathrm{f}}_{jit} - q_{jit}\left[\zeta\left(\frac{z^{\mathrm{he}}_{it}}{v^{\mathrm{h}}_{jit}}, \frac{y^{\mathrm{w}}_{it}}{v^{\mathrm{h}}_{jit}}\right) + \theta\right]\right\}; \tag{1.16}$$

$$\lim_{t\to\infty} \exp^{-\int_0^t r_v dv} q_{jit} N_{jit} g(\Lambda_i) = 0. \tag{1.17}$$

The equations represented by (1.14) characterize the optimal number of new hires. In the case arising in the steady-state analysis below, the shadow

[2] The quit rate function has the following first derivatives: $\zeta_1 > 0$ and $\zeta_2 > 0$. By virtue of the firm's second-order condition for maximization, $\zeta_{11} > 0$ and $\zeta_{22} > 0$. We also make the assumption that an increase in the non-wage income raises a worker's marginal propensity to quit with respect to wage prospects elsewhere, that is, $\zeta_{12} > 0$.

[3] The flow of output at firm j is then given by $\int_{\underline{\Lambda}}^{\infty} \Lambda_i[1 - \beta h_{jit}] N_{jit} g(\Lambda_i) d\Lambda_i$.

value of a trained worker is equal to the marginal training cost in output terms. Equation (1.15) gives the optimal tradeoff between real wage and turnover cost, equating the marginal cost of raising v_i^{f} to the marginal benefit. Equation (1.16) relates the shadow value of functional employees to the total marginal benefit of having one more employee. The transversality condition is in (1.17). These equations summarize the conditions that have to be satisfied for the typical firm.

To move to the equilibrium conditions, we use the Salop–Calvo approximation for z_{it}^{he}, namely, $z_{it}^{\mathrm{he}} = N_{it}^{\mathrm{e}} v_{it}^{\mathrm{he}}$. (Using the exit rate from the unemployment pool would not differ in the steady state.) On any equilibrium (correct expectations) path with identical firms, $v_{jit}^{\mathrm{h}} = v_{it}^{\mathrm{h}} = v_{it}^{\mathrm{he}}$ and $N_{jit} = 1 - u_{it} \equiv N_{it} = N_{it}^{\mathrm{e}}$. Hence we obtain a subsystem of equations in the equilibrium path of the economy. For any exogenously given path of the instantaneous real interest rates, this subsystem is

$$\dot{q}_{it} = q_{it}\left[\zeta\left(N_{it}, \frac{y_{it}^{\mathrm{w}}}{v_{it}^{\mathrm{h}}}\right) + \theta + r_t\right] - \left[\Lambda_i - v_{it}^{\mathrm{f}}\right]; \tag{1.18}$$

$$\dot{N}_{it} = N_{it}\left[h_{it} - \zeta\left(N_{it}, \frac{y_{it}^{\mathrm{w}}}{v_{it}^{\mathrm{h}}}\right) - \theta\right]; \tag{1.19}$$

$$N_{it}\left\{-1 + q_{it}\left[\left(\frac{N_{it}}{v_{it}^{\mathrm{h}}}\right)\zeta_1 + \left(\frac{y_{it}^{\mathrm{w}}}{v_{it}^{\mathrm{h}2}}\right)\zeta_2\right]\frac{dv_{it}^{\mathrm{h}}}{dv_{it}^{\mathrm{f}}}\right\} = 0. \tag{1.20}$$

3 Open-economy incidence of tax subsidy schemes

In steady state, $\dot{N}_{it} = 0$. This and (1.19) give the steady-state employment (SSE) condition that hires balance quits and mortality:

$$h_i = \zeta\left(N_{it}, \frac{y_{it}^{\mathrm{w}}}{v_{it}^{\mathrm{h}}}\right) + \theta. \tag{1.21}$$

This implies that $q_i = \Lambda_i \beta$.

With $\dot{q}_{it} = 0$ in (1.18) and $q_i = \Lambda_i\beta$, the zero-profit (ZP) condition that quasi-rents cover interest and depreciation on training becomes

$$\frac{v_i^{\mathrm{f}}}{\Lambda_i} = 1 - \beta\left[\zeta\left(N_i, \frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}}\right) + \theta + r^*\right], \tag{1.22}$$

where r^* is substituted for the domestic interest rate. Since quitting is increasing in N_i and y_i^{w}, the zero-profit wage must be decreasing in those variables.

Assuming that the employment rate is always strictly positive, we obtain from (1.20) the incentive-wage (IW) condition for the hourly compensation that minimizes compensation plus training cost. The cost per employee of paying a penny more in annual wages is one. The cost saving, or benefit, per employee of doing so is the opportunity cost of replacing each defector, $\beta\Lambda_i$, times the number of annual quits per employee that would be saved. Equating these two gives

$$1 = \beta\Lambda_i \left[N_i \zeta_1 + \left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}} \right) \zeta_2 \right] \left[\left(\frac{1}{v_i^{\mathrm{h}}} \right) \left(\frac{dv_i^{\mathrm{h}}}{dv_i^{\mathrm{f}}} \right) \right]. \tag{1.23}$$

The flat (constant) subsidy case gives

$$\begin{aligned} 1 &= \beta\Lambda_i \left[N_i \zeta_1 + \left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}} \right) \zeta_2 \right] \left[\frac{1}{1+\tau} \bigg/ \frac{v_i^{\mathrm{f}} + s^{\mathrm{F}}}{1+\tau} \right] \\ &= \beta\Lambda_i \left[N_i \zeta_1 + \left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}} \right) \zeta_2 \right] \left[\frac{1}{v_i^{\mathrm{f}} + s^{\mathrm{F}}} \right], \end{aligned} \tag{1.24}$$

since $dv_i^{\mathrm{h}}/dv_i^{\mathrm{f}} \equiv 1/(1+\tau)$ and $v_i^{\mathrm{h}} \equiv (v_i^{\mathrm{f}} + s^{\mathrm{F}})/(1+\tau)$. The graduated subsidy case gives

$$1 = \beta\Lambda_i \left[N_i \zeta_1 + \left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}} \right) \zeta_2 \right] \left[\frac{1 + S'\left(v_i^{\mathrm{f}}\right)}{v_i^{\mathrm{f}} + S\left(v_i^{\mathrm{f}}\right)} \right]. \tag{1.25}$$

Notice that (1.25) can be satisfied as an equality only if $|S'(v_i^{\mathrm{f}})| < 1$. If $|S'(v_i^{\mathrm{f}})| > 1$, each firm would find it profitable to drive the wage all the way down in order to gain a higher subsidy.

The third general equilibrium condition arises from the firms' assets. The assets are the investments in their employees, the ownership claims to which – the equity shares – generate non-wage income and have an equilibrium value. As before, we use the Blanchard–Yaari setup to generate, in steady state, the equation:

$$r^* = \rho + \frac{\theta}{1 + \left(\frac{v_i^{\mathrm{h}}}{y_i^{\mathrm{w}}} \right) N_i}. \tag{1.26}$$

This condition makes the non-wage-income-to-wage ratio an implicit function of the unemployment rate and of the interest rate:

$$\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}} = \Omega(r^* - \rho, N_i), \;\; \Omega_1 > 0, \Omega_2 > 0. \tag{1.27}$$

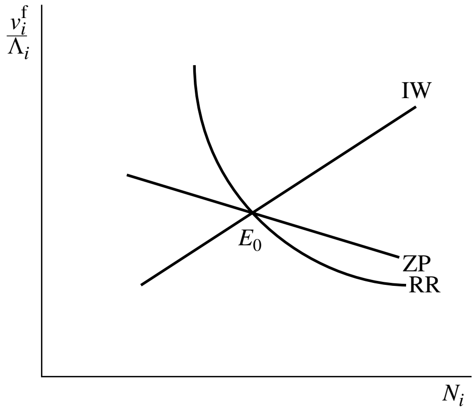

Figure 1.2 *Labor, product and capital market equilibrium*

3.1 Long-run effects of the flat subsidy

Substituting (1.27) into (1.22) and (1.24) gives the reduced-form system in the flat subsidy case:

$$\frac{v_i^{\mathrm{f}}}{\Lambda_i} = 1 - \beta[\zeta(N_i, \Omega(r^* - \rho, N_i)) + \theta + r^*], \tag{1.28}$$

$$\frac{v_i^{\mathrm{f}}}{\Lambda_i} + \frac{s^{\mathrm{F}}}{\Lambda_i} = \beta[N_i\zeta_1(N_i, \Omega(r^* - \rho, N_i)) + \Omega(r^* - \rho, N_i)\zeta_2(N_i, \Omega(r^* - \rho, N_i))]. \tag{1.29}$$

Suppose that initially the ad valorem payroll tax rate is zero and the subsidy is also zero. Equation (1.28) can be represented as a downward-sloping zero-profit schedule and (1.29) can be depicted as an upward-sloping wage curve in the Marshallian plane shown in figure 1.2. Examining (1.26), and recalling that in the absence of the tax-subsidy scheme $v_i^{\mathrm{h}} \equiv v_i^{\mathrm{f}}$, notice that we can also draw a family of hyperbolas in figure 1.2 with each hyperbola lying north-east corresponding to a higher level of y_i^{w}. Note also that when the ZP curve cuts the hyperbola from below, as we have drawn in figure 1.2, the labor cost elasticity of labor demand is implied to exceed one. (In that case, as we shall see, the proportionate increase of N_i effected by the subsidy exceeds the proportionate decrease of $v_i^{\mathrm{f}}/\Lambda_i$ that the increased N_i induces so that, on balance, the product $(v_i^{\mathrm{f}}/\Lambda_i)\,N_i$ is up.) The algebraic slope of the zero-profit curve is given by $-\beta[\zeta_1 + \zeta_2\Omega_2]$, which, in the absence of any other factors leading to diminishing returns to labor, depends only on the sensitivity of the quit function to the economy-wide rate of employment (or unemployment). The zero-profit curve slopes downward both because a lower rate of unemployment implies a tighter labor market, which induces higher quits, and because it implies a higher non-wage income-to-wage ratio, which also raises the propensity to quit. For the United States over the period 1931–1962, Eagly (1965) obtains an estimate of the elasticity of the quit rate with respect to the unemployment rate that is equal to -0.634.[4] If we accept that, in the equilibrium steady-state scenario we are considering, the quit rate does not vary much with movements in the employment rate, the zero-profit curve will be somewhat flat, that is, the labor cost elasticity of the zero-profit curve will be high. We also notice that the same diagram (figure 1.2) represents the equilibrium for every type-i worker. The employment rate, N_i, the real effective wage, $v_i^{\mathrm{f}}/\Lambda_i$, and the non-wage income taken as a ratio to productivity level, $y_i^{\mathrm{w}}/\Lambda_i$, are the same for every type-i worker, so the real wage, v_i^{f}, is twice as high for a worker who is twice as productive as another worker.[5] The non-wage income, y_i^{w}, corresponding to the hyperbola passing through E_0 is also twice as high for a worker who is twice as productive as another worker.

Consider now the long-run employment effects of a flat (constant) subsidy. The derivative of N_i with respect to s^{F} is calculated to be

$$\frac{dN_i}{ds^{\mathrm{F}}} = \frac{\Lambda_i^{-1}}{\beta[2(\zeta_1 + \zeta_2\Omega_2) + N_i(\zeta_{11} + \zeta_{12}\Omega_2) + \Omega(\zeta_{21} + \zeta_{22}\Omega_2)]} > 0 \tag{1.30}$$

for every type i. The argument that this inequality is unambiguously positive is the following. Assume that there was no change in unemployment so that we were at an unchanged point (N_i, v_i^{f}) on the zero-profit curve and firms have returned to the original point that they were at before. The proportional payroll tax, taken by itself, has two effects. First, a penny

[4] Looking at the effects of wage differentials on quits, Krueger and Summers (1988: 280) find that "at the mean the elasticity of quits with respect to the wage premium is $-.07/.26 = -.27$." They reason that, taken together, "these results imply that a 10 per cent increase in the wage differential brings about a .3 per cent increase in output through reduced quits alone. This suggests that although turnover does adversely affect output, reductions in turnover alone are not sufficient to justify wage premiums of the magnitude actually observed unless fixed costs of hiring are very high or labor's share in output is very low."

[5] The equalization of unemployment rate result depends on the assumption that the marginal training cost in *manhours*, β, is the same across all types of workers. If we have $\beta_i > \beta_j$, then it can be shown that the unemployment rate for type-i workers will be higher than that for type-j workers. Note that this assumption is consistent with $\Lambda_i\beta_i < \Lambda_j\beta_j$, that is, although the marginal training cost for type-i workers is higher when measured in manhours, it could be lower when measured in terms of output on account of its lower productivity.

increase in v_i^{f} increases v_i^{h} by only a fraction of a penny, namely, $1/(1+\tau)$. This lowers the marginal benefit of a penny increase in v_i^{f}. Second, the proportional payroll tax lowers v_i^{h} and, under correct expectations, v_i^{he}, in the same proportion for every type i. With the employment rate unchanged, y_i^{w} would also be reduced by the same proportion. Then each additional penny received by an employee now has a greater impact on v_i^{h} taken as a ratio to expected real wage earnings elsewhere and taken as a ratio to non-wage income, so that the salutary effect on quitting is increased. Through this channel the marginal benefit of a penny increase in v_i^{f} is increased. If, instead of financing the subsidy, the proceeds from the payroll tax were, say, thrown into the sea, the two effects would exactly cancel out, leaving employment unaffected. There is, however, a third effect arising from the presence of the constant subsidy. In the presence of the subsidy, an additional penny received by an employee has a smaller impact on $v_i^{\mathrm{h}}/v_i^{\mathrm{he}}$ and $v_i^{\mathrm{h}}/y_i^{\mathrm{w}}$, so that the salutary effect on quitting is reduced. In the general equilibrium involving correct expectations and long-run capital market equilibrium, the incentive-wage condition can be written as

$$1 = \beta\Lambda_i[N_i\zeta_1 + \Omega(r^* - \rho, N_i)\zeta_2]\left[\frac{1}{1+\tau}\bigg/\frac{v_i^{\mathrm{f}} + s^{\mathrm{F}}}{1+\tau}\right].$$

We can see from the righthand side of this equation that the two effects arising from the presence of $(1+\tau)$ exactly cancel out. This implies that in the long run, after wealth has fully adjusted, the payroll tax is neutral for employment. It follows that, if a penny increase in v_i^{f} had a marginal benefit equal to marginal cost at the original employment rate, it must now have a marginal benefit less than the marginal cost. Hence firms cut their v_i^{f} and employment is expanded as a result.

We can see that, with the same dollar amount of wage subsidy given to each type-i worker, less productive workers enjoy a higher subsidy relative to their productivity level. In figure 1.3 we show that the employment effect is larger for less productive workers as their wage curve is shifted further down than that for more productive workers.

Consider now the long-run wage effects of the flat subsidy. We note that, under a balanced-budget policy, the following relationship holds:

$$\int_{\underline{\Lambda}}^{\infty} s^{\mathrm{F}} N_i g(\Lambda_i)d\Lambda_i = \int_{\underline{\Lambda}}^{\infty} \tau v_i^{\mathrm{h}} N_i g(\Lambda_i)d\Lambda_i.$$

As noted earlier, around an equilibrium with no tax subsidy, N_i is equal for every type i. It follows that the budget constraint can be simplified to $\tau = s^{\mathrm{F}}/v_{\mathrm{mean}}^{\mathrm{h}}$, where $v_{\mathrm{mean}}^{\mathrm{h}} \equiv \int_{\underline{\Lambda}}^{\infty} v_i^{\mathrm{h}} g(\Lambda_i)d\Lambda_i$. Using this, and noting that around a zero tax-subsidy equilibrium $(v_i^{\mathrm{h}}/v_{\mathrm{mean}}^{\mathrm{h}}) = (\Lambda_i/\Lambda_{\mathrm{mean}})$, it

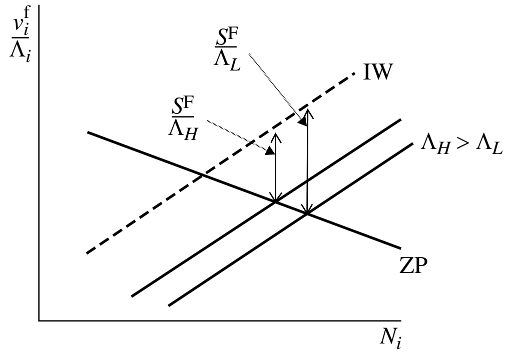

Figure 1.3 *Effects of a flat subsidy*

is straightforward to show that

$$\left.\frac{dv_i^{\mathrm{h}}}{ds^{\mathrm{F}}}\right|_{\tau=0} = \frac{\eta_{\mathrm{ZP}}}{\eta_{\mathrm{ZP}} + \eta_{\mathrm{IW}}} - \frac{\Lambda_i}{\Lambda_{\mathrm{mean}}}, \tag{1.31}$$

where η_{ZP} and η_{IW} are the elasticities of the zero-profit and wage curves, respectively. For a worker whose Λ_i is sufficiently low, say $\Lambda_i \to \underline{\Lambda} \to 0$, the derivative of v_i^{h} with respect to s^{F} is unambiguously positive. But employment is expanded everywhere.

3.2 *Short-run effects of the flat subsidy*

Consider now briefly the short run in which wealth and y_i^{w} are given. Here the subsidy provides an additional boost to employment. With net wealth and interest unchanged, the increased take-home pay leads workers to value their job more highly. This has the effect, at any employment rate, of raising the firm's real demand wage as the propensity to quit is reduced. Around a zero tax-subsidy equilibrium, the vertical shift of the iso-y_i^{w} ZP curve is given by

$$\left.\frac{dv_i^{\mathrm{f}}}{ds^{\mathrm{F}}}\right|_{\mathrm{ZP}} = \frac{\beta\Lambda_i\zeta_2\left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}2}}\right)\left[1 - \frac{\Lambda_i}{\Lambda_{\mathrm{mean}}}\right]}{1 - \beta\Lambda_i\zeta_2\left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}2}}\right)},$$

which is positive for any worker whose productivity is below the mean.[6] The decreased propensity to quit on account of the reduced non-wage income relative to wage ratio also has the effect of shifting down the

[6] Around a zero tax-subsidy equilibrium, $1 - \beta\Lambda_i\zeta_2(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}2}}) = \beta\Lambda_i N_i/v_i^{\mathrm{h}} > 0$.

incentive-wage curve, which is on top of the shift owing to the wedge caused by the subsidy. The vertical shift of the iso-y_i^{w} IW curve is given by

$$\left.\frac{dv_i^{\mathrm{f}}}{ds^{\mathrm{F}}}\right|_{\mathrm{IW}} = \frac{-1-\beta\Lambda_i\left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}2}}\right)\left[\zeta_2+N_i\zeta_{12}+\left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}}\right)\zeta_{22}\right]\left[1-\frac{\Lambda_i}{\Lambda_{\mathrm{mean}}}\right]}{1+\beta\Lambda_i\left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}2}}\right)\left[\zeta_2+N_i\zeta_{12}+\left(\frac{y_i^{\mathrm{w}}}{v_i^{\mathrm{h}}}\right)\zeta_{22}\right]},$$

which is unambiguously negative for a worker whose productivity level is below the mean. From (1.26) we see that, at given N_i, the non-wage income, y_i^{w}, is increased by the same proportion as the rise in v_i^{h} for the low-wage worker. Hence, in the long run, wealth accumulation ultimately shifts the ZP curve back to its original position and the IW curve also shifts up as wealth catches up to the increased take-home pay. However, a wedge remains, implying that employment is expanded throughout the distribution in the long run, as shown earlier. For low-wage workers, there is an additional boost to employment in the short run.

3.3 *Long-run effects of the graduated subsidy*

Now the graduated subsidy: Equation (1.29) is replaced by

$$\frac{v_i^{\mathrm{f}}+S(v_i^{\mathrm{f}})}{\Lambda_i[1+S'(v_i^{\mathrm{f}})]} = \beta[N_i\zeta_1(N_i,\Omega(r^*-\rho,N_i)) \\ +\Omega(r^*-\rho,N_i)\zeta_2(N_i,\Omega(r^*-\rho,N_i))]. \quad (1.32)$$

Around a zero tax-subsidy equilibrium, the response of N_i to a small change in $s^* \equiv S(v_i^{\mathrm{f}*})$ is then calculated to be

$$\left.\frac{dN_i}{ds^*}\right|_{\tau=0} \\ = \frac{\Lambda_i^{-1}+\Lambda_i^{-1}\left\{\frac{[v_i^{\mathrm{f}*}S''/(1+S')]\tilde{\eta}_{\mathrm{IW}}}{(1-[v_i^{\mathrm{f}*}S''/(1+S')])\tilde{\eta}_{\mathrm{IW}}+\eta_{\mathrm{ZP}}}\right\}}{(1+S')\beta[(\zeta_1+\zeta_2\Omega_2)+N_i(\zeta_{11}+\zeta_{12}\Omega_2)+\Omega(\zeta_{21}+\zeta_{22}\Omega_2)]+\beta[\zeta_1+\zeta_2\Omega_2]}, \quad (1.33)$$

where

$$\tilde{\eta}_{\mathrm{IW}} = \frac{\Lambda_i\beta[(\zeta_1+\zeta_2\Omega_2)+N_i(\zeta_{11}+\zeta_{12}\Omega_2)+\Omega(\zeta_{21}+\zeta_{22}\Omega_2)]N_i}{v_i^{\mathrm{f}}/(1+S')} > 0.$$

Expressing $\eta_{\mathrm{IW}} \equiv \{(1+S')-[v_i^{\mathrm{f}}S''/(1+S')]\}\tilde{\eta}_{\mathrm{IW}}$, the condition that the wage curve be positively sloped in the (N_i, v_i^{f}) plane is that $S'' < (1+S')^2/v_i^{\mathrm{f}}$. Given the restriction that $|S'(v_i^{\mathrm{f}})| < 1$, a sufficient condition for a graduated subsidy scheme paying $s^* = s^{\mathrm{F}}$ to give an extra boost to

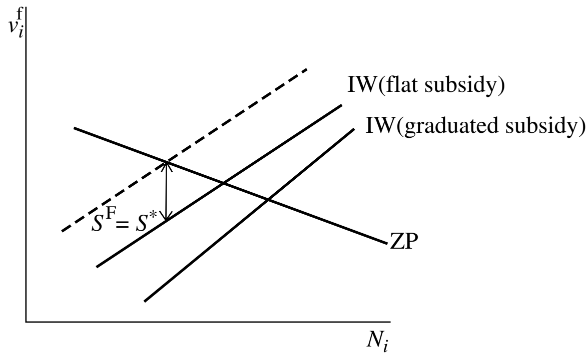

Figure 1.4 *Comparison of flat and graduated subsidies*

employment is therefore that $0 < S'' < (1 + S')^2/v_i^{\mathrm{f}}$. With graduation, there are two effects at work when compared with the constant subsidy case, as shown in figure 1.4. First, with graduation firms are induced to moderate wage rates above the bottom in order to gain a larger subsidy. For $s^* = s^{\mathrm{F}}$, figure 1.4 shows that the wage curve is shifted further down under a graduated scheme. Second, graduation changes the slope of the wage curve. Whereas a constant subsidy scheme has no effect on the slope of the wage curve (there being a parallel shift), with a graduated scheme the new wage curve becomes steeper at higher wages. The restriction on S'' is sufficient to ensure that the "shift" as well as the "slope" effects of graduation give a bigger boost to employment compared with the constant subsidy case. Note also that, by designing a subsidy plan such that the subsidy asymptotically reaches zero as v_i^{f} is increased, we ensure that employment is raised throughout the distribution, although the expansionary effect is smaller at higher v_i^{f}.

Consider now the long-run wage effects. We can show that, around a zero tax-subsidy equilibrium, the following derivative holds:

$$\left.\frac{dv_i^{\mathrm{h}}}{ds^*}\right|_{\tau=0} = \left\{ \frac{\eta_{\mathrm{ZP}} - \left[\frac{v_i^{\mathrm{f}*} S''}{1+S'}\right]\tilde{\eta}_{\mathrm{IW}}}{\left(1 - \left[\frac{v_i^{\mathrm{f}*} S''}{1+S'}\right]\right)\tilde{\eta}_{\mathrm{IW}} + \eta_{\mathrm{ZP}}} \right\} - \left[\frac{\Lambda_i}{\Lambda_{\mathrm{mean}}}\right]\left[\frac{d\mathcal{S}}{ds^*}\right], \tag{1.34}$$

where $\mathcal{S} \equiv \int_{\Lambda}^{\infty} S(v_i^{\mathrm{f}}) g(\Lambda_i) d\Lambda_i$ and $d\mathcal{S}/ds^* > 0$. If we further restrict the value of S'' such that $0 < S'' < (1 + S')^2/v_i^{\mathrm{f}} - (\eta_{IW}/\eta_{ZP})(1 + S')/v_i^{\mathrm{f}}$, the first curly brace term in (1.34) is unambiguously positive. Notice from (1.33) that employment is increasing in S''. If we strike a balance in our choice of S'' with regard to the extra expansionary employment effect on the one hand and the wage effect on the other hand, we can obtain a higher take-home wage for a worker whose Λ_i is sufficiently low along with higher employment.

3.4 Long-run effects of a hiring subsidy

Before concluding our analysis of the small open economy, let us examine the effects of a hiring subsidy in our model. Suppose that an ad valorem payroll tax is used to finance a flat hiring subsidy of s^{HF} for each new recruit hired. It is straightforward to show that our two fundamental equations giving the reduced-form ZP and IW schedules become, respectively,

$$\frac{v_i^{\mathrm{f}}}{\Lambda_i} = 1 - \left[\beta - \frac{s^{\mathrm{HF}}}{\Lambda_i}\right][\zeta(N_i, \Omega(r^* - \rho, N_i)) + r^* + \theta]; \qquad (1.35)$$

$$\frac{v_i^{\mathrm{f}}}{\Lambda_i} = \left[\beta - \frac{s^{\mathrm{HF}}}{\Lambda_i}\right][N_i\zeta_1(N_i, \Omega(r^* - \rho, N_i)) + \Omega(r^* - \rho, N_i)\zeta(N_i, \Omega(r^* - \rho, N_i))]. \qquad (1.36)$$

Such a policy shifts the ZP curve up but shifts the IW curve down, leading to an unambiguous expansion of equilibrium employment but possible decline of the product wage, v_i^{f}. (In contrast, under both the flat and graduated subsidy plans, the before-tax wage of the workers, $v_i^{\mathrm{f}} + s_i$, unambiguously rises.) The take-home wage would accordingly fall further as the payroll tax is applied, though this must be set against the subsidy that each new recruit receives when hired. We obtain the following derivative:[7]

$$\left.\frac{d[v_i^{\mathrm{f}}/(1+\tau) + (r^* + \theta)s^{\mathrm{HF}}]}{ds^{\mathrm{HF}}}\right|_{s^{\mathrm{HF}}=0} = (\zeta + \theta)\left[\mu - \left(\frac{\Lambda_i}{\Lambda_{\mathrm{mean}}}\right)\right] + (1+\mu)r^* + \theta - (1-\mu)[N_i\zeta_1 + \Omega\zeta_2],$$

where

$$0 < \mu \equiv \frac{(\zeta_1 + \zeta_2\Omega_2) + (\zeta_{11} + \zeta_{12}\Omega_2)N_i + (\zeta_{21} + \zeta_{22}\Omega_2)\Omega}{2(\zeta_1 + \zeta_2\Omega_2) + (\zeta_{11} + \zeta_{12}\Omega_2)N_i + (\zeta_{21} + \zeta_{22}\Omega_2)\Omega} < 1.$$

[7] The balanced-budget condition with a hiring subsidy simplifies to $\tau = [(\zeta + \theta)s^{\mathrm{HF}}/v^{\mathrm{h}}_{\mathrm{mean}}]$ around a zero hiring subsidy equilibrium, noting that in the steady state the hiring rate equals $\zeta + \theta$ for every type of worker.

4 Closed economy incidence

We confine our analysis to a flat subsidy in the closed economy financed by a proportional payroll tax. For any r, our reduced-form ZP and IW curves are written respectively as

$$\frac{v_i^{\mathrm{f}}}{\Lambda_i} = 1 - \beta[\zeta(N_i, \Omega(r - \rho, N_i)) + \theta + r], \tag{1.37}$$

$$\begin{aligned}\frac{v_i^{\mathrm{f}}}{\Lambda_i} + \frac{s^{\mathrm{F}}}{\Lambda_i} = \beta[N_i\zeta_1(N_i, \Omega(r - \rho, N_i)) \\ + \Omega(r - \rho, N_i)\zeta_2(N_i, \Omega(r - \rho, N_i))],\end{aligned} \tag{1.38}$$

where we have again substituted for $y_i^{\mathrm{w}}/v_i^{\mathrm{h}}$ the function $\Omega(r - \rho, N_i)$ obtained from the Blanchardian relationship expressed as

$$r = \rho + \frac{\theta}{1 + \left(v_i^{\mathrm{h}}/y_i^{\mathrm{w}}\right) N_i}. \tag{1.39}$$

We note from (1.37) and (1.38) that, by equating the required incentive wage to the demand wage, we can express the employment rate of any type-i worker as an implicit function of the interest rate and the subsidy relative to productivity level, namely,

$$N_i = \epsilon(r; (s^{\mathrm{F}}/\Lambda_i)); \ \epsilon_1 < 0; \ \epsilon_2 > 0. \tag{1.40}$$

The function ϵ is interpretable as the demand for the stock of employees in steady state. The value of the total stock of employees, which are the only form of asset in the closed economy, is $A \equiv \int_{\underline{\Lambda}}^{\infty} \beta\Lambda_i N_i g(\Lambda_i) d\Lambda_i$ because each employee is worth $\beta\Lambda_i$. By (1.40), A is a decreasing function of the rate of interest:

$$A = \int_{\underline{\Lambda}}^{\infty} \beta\Lambda_i \epsilon(r; (s^{\mathrm{F}}/\Lambda_i)) g(\Lambda_i) d\Lambda_i. \tag{1.41}$$

An expression for the average supply of wealth per member of the type-i workforce is obtained from (1.39) as

$$W_i = \left(\frac{v_i^{\mathrm{h}} N_i}{r + \theta}\right)\left[\frac{r - \rho}{\theta + \rho - r}\right]. \tag{1.42}$$

As before, excluding the case where $r - \rho > \theta$, we have a well-defined steady state, with the righthand side of (1.42) being unambiguously positive. The total supply of wealth per worker, under a balanced budget, is given by

$$W = \left[\frac{r - \rho}{(\theta + \rho - r)(r + \theta)}\right] \int_{\underline{\Lambda}}^{\infty} v_i^{\mathrm{f}} N_i g(\Lambda_i) d\Lambda_i. \tag{1.43}$$

Further, using (1.37) and (1.40) in (1.43), we obtain an expression giving us the total desired supply of wealth as a function of the rate of interest:

$$W = \left[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}\right] \int_{\underline{\Lambda}}^{\infty} \left\{1 - \beta\left[\zeta\left(\epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right)\right),\right.\right.$$
$$\left.\left.\Omega\left(r-\rho, \epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right)\right) + r + \theta\right]\right\} \epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right) \Lambda_i g(\Lambda_i) d\Lambda_i. \tag{1.44}$$

Suppose that initially the subsidy and payroll tax are zero. In that case, we note from (1.37) and (1.38) that setting $s^{\mathrm{F}} = 0$ implies that N_i and $y_i^{\mathrm{w}}/v_i^{\mathrm{h}}$ are equal across all types of workers. Consequently, the quit rate is initially identical across all types of workers. As in our earlier discussion in the neoclassical case, we can argue that the per worker supply of wealth is upward sloping initially, but at very high r may bend backward as in figure 1.1.[8] In the same plane, per worker demand for the domestic assets in value terms is downward sloping. We suppose that the equilibrium r is unique or that only the lowest equilibrium r is empirically relevant.

To see how the tax-subsidy policy affects the rate of interest, it will help to have a sharper characterization of this equilibrium. Since the quit rate is equal across all types of workers in the neighborhood of the zero-subsidy equilibrium, we can simplify the equilibrium condition to

$$\begin{aligned}
W \equiv & \left[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}\right] \\
& \times \left\{1 - \beta\left[\zeta\left(\epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right)\right), \Omega\left(r-\rho, \epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right)\right) + r + \theta\right]\right\} \\
& \times \int_{\underline{\Lambda}}^{\infty} \epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right) \Lambda_i g\left(\Lambda_i\right) d\Lambda_i \\
= & \ \beta \int_{\underline{\Lambda}}^{\infty} \epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right) \Lambda_i g(\Lambda_i) d\Lambda_i \equiv A.
\end{aligned} \tag{1.45}$$

The equilibrium r is therefore given by

$$\left[\frac{r-\rho}{(\theta+\rho-r)(r+\theta)}\right] \left\{1 - \beta\left[\zeta\left(\epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right)\right),\right.\right.$$
$$\left.\left.\Omega\left(r-\rho, \epsilon\left(r;\frac{s^{\mathrm{F}}}{\Lambda_i}\right)\right) + r + \theta\right]\right\} = \beta. \tag{1.46}$$

[8] Although the increase in r leads to a decline in the real demand wage, the fall in N_i acts to lower the quit propensity and hence indirectly acts to offset the fall in wage. We assume that the direct effect dominates.