Economic Dynamics

Phase Diagrams and Their Economic Application

Second Edition

This is the substantially revised and restructured second edition of Ron Shone's successful undergraduate and graduate textbook *Economic Dynamics.*

The book provides detailed coverage of dynamics and phase diagrams including: quantitative and qualitative dynamic systems, continuous and discrete dynamics, linear and nonlinear systems and single equation and systems of equations. It illustrates dynamic systems using *Mathematica*, *Maple* and spreadsheets. It provides a thorough introduction to phase diagrams and their economic application and explains the nature of saddle path solutions.

The second edition contains a new chapter on oligopoly and an extended treatment of stability of discrete dynamic systems and the solving of first-order difference equations. Detailed routines on the use of *Mathematica* and *Maple* are now contained in the body of the text, which now also includes advice on the use of *Excel* and additional examples and exercises throughout. The supporting website contains a solutions manual and learning tools.

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Preface to the second edition

I was very encouraged with the reception of the first edition, from both staff and students. Correspondence eliminated a number of errors and helped me to improve clarity. Some of the new sections are in response to communications I received.

The book has retained its basic structure, but there have been extensive revisions to the text. Part I, containing the mathematical background, has been considerably enhanced in all chapters. All chapters contain new material. This new material is largely in terms of the mathematical content, but there are some new economic examples to illustrate the mathematics. Chapter 1 contains a new section on dimensionality in economics, a much-neglected topic in my view. Chapter 3 on discrete systems has been extensively revised, with a more thorough discussion of the stability of discrete dynamical systems and an extended discussion of solving second-order difference equations. Chapter 5 also contains a more extensive discussion of discrete systems of equations, including a more thorough discussion of solving such systems. Direct solution methods using *Mathematica* and *Maple* are now provided in the main body of the text. Indirect solution methods using the Jordan form are new to this edition. There is also a more thorough treatment of the stability of discrete systems.

The two topics covered in chapter 6 of the first edition have now been given a chapter each. This has allowed topics to be covered in more depth. Chapter 6 on control theory now includes the use of *Excel's Solver* for solving discrete control problems. Chapter 7 on chaos theory has also been extended, with a discussion of Sarkovskii's theorem. It also contains a much more extended discussion of bifurcations and strange attractors.

Changes to part II, although less extensive, are quite significant. The mathematical treatment of cobwebs in chapter 8 has been extended and there is now a new section on stock models and another on chaotic demand and supply. Chapter 9 on dynamic oligopoly is totally new to this edition. It deals with both discrete and continuous dynamic oligopoly and goes beyond the typical duopoly model. There is also a discussion of an R&D dynamic model of duopoly and a brief introduction to Schumpeterian dynamics. Chapter 11 now includes a discussion of deflationary 'death spirals' which have been prominent in discussions of Japan's downturn. Cagan's model of hyperinflations is also a new introduction to this chapter.

The open economy was covered quite extensively in the first edition, so these chapters contain only minor changes. Population models now include a consideration of age classes and Leslie projection matrices. This material is employed

in chapter 15 to discuss culling policy. The chapter on overlapping generations modelling has been dropped in this edition to make way for the new material. Part of the reason for this is that, as presented, it contained little by the way of dynamics. It had much more to say about nonlinearity.

Two additional changes have been made throughout. *Mathematica* and *Maple* routines are now generally introduced into the main body of the text rather than as appendices. The purpose of doing this is to show that these programmes are 'natural' tools for the economist. Finally, there has been an increase in the number of questions attached to almost all chapters. As in the first edition, the full solution to all these questions is provided on the Cambridge University website, which is attached to this book: one set of solutions provided in *Mathematica* notebooks and an alternative set of solutions provided in *Maple* worksheets.

Writing a book of this nature, involving as it does a number of software packages, has become problematic with constant upgrades. This is especially true with *Mathematica* and *Maple*. Some of the routines provided in the first edition no longer work in the upgrade versions. Even in the final stages of preparing this edition, new upgrades were occurring. I had to make a decision, therefore, at which upgrade I would conclude. All routines and all solutions on the web site are carried out with *Mathematica 4* and *Maple 6*.

I would like to thank all those individuals who wrote or emailed me on material in the first edition. I would especially like to thank Mary E. Edwards, Yee-Tien Fu, Christian Groth, Cars Hommes, Alkis Karabalis, Julio Lopez-Gallardo, Johannes Ludsteck and Yanghoon Song. I would also like to thank Simon Whitby for information and clarification on new material in chapter 9. I would like to thank Ashwin Rattan for his continued support of this project and Barbara Docherty for an excellent job of copy-editing, which not only eliminated a number of errors but improved the final project considerably.

The author and publishers wish to thank the following for permission to use copyright material: Springer-Verlag for the programme listing on p. 192 of *A First Course in Discrete Dynamic Systems* and the use of the Visual D Solve software package from *Visual D Solve*; Cambridge University Press for table 3 from *British Economic Growth 1688–1959*, p. 8.

The publisher has used its best endeavours to ensure that the URLs for external websites referred to in this book are correct and active at the time of going to press. However, the publisher has no responsibility for the websites and can make no guarantee that a site will remain live or that the content is or will remain appropriate.

March 2002

Preface to the first edition

The conception of this book began in the autumn semester of 1990 when I undertook a course in Advanced Economic Theory for undergraduates at the University of Stirling. In this course we attempted to introduce students to dynamics and some of the more recent advances in economic theory. In looking at this material it was quite clear that phase diagrams, and what mathematicians would call qualitative differential equations, were becoming widespread in the economics literature. There is little doubt that in large part this was a result of the rational expectations revolution going on in economics. With a more explicit introduction of expectations into economic modelling, adjustment processes became the mainstay of many economic models. As such, there was a movement away from models just depicting comparative statics. The result was a more explicit statement of a model's dynamics, along with its comparative statics. A model's dynamics were explicitly spelled out, and in particular, vectors of forces indicating movements when the system was not in equilibrium. This led the way to solving dynamic systems by employing the theory of differential equations. Saddle paths soon entered many papers in economic theory. However, students found this material hard to follow, and it did not often use the type of mathematics they were taught in their quantitative courses. Furthermore, the material that was available was very scattered indeed.

But there was another change taking place in Universities which has a bearing on the way the present book took shape. As the academic audit was about to be imposed on Universities, there was a strong incentive to make course work assessment quite different from examination assessment. Stirling has always had a long tradition of course work assessment. In the earlier period there was a tendency to make course work assessment the same as examination assessment: the only real difference being that examinations could set questions which required greater links between material since the course was by then complete. In undertaking this new course, I decided from the very outset that the course work assessment would be quite different from the examination assessment. In particular, I conceived the course work to be very 'problem oriented'. It was my belief that students come to a better understanding of the economics, and its relation to mathematics, if they carry out problems which require them to explicitly solve models, and to go on to discuss the implications of their analysis.

This provided me with a challenge. There was no material available of this type. Furthermore, many economics textbooks of an advanced nature, and certainly the

published articles, involved setting up models in general form and carrying out very tedious algebraic manipulations. This is quite understandable. But such algebraic manipulation does not give students the same insight it may provide the research academic. A compromise is to set out models with specific numerical coefficients. This has at least four advantages.

It allowed the models to be *solved explicitly.* This means that students can get to grips with the models themselves fairly quickly and easily.

Generalisation can always be achieved by replacing the numerical coefficients by *unspecified parameters.* Or alternatively, the models can be solved for different values, and students can be alerted to the fact that a model's solution is quite dependent on the value (sign) of a particular parameter.

The dynamic nature of the models can more readily be *illustrated*. Accordingly concentration can be centred on the economics and not on the mathematics.

Explicit solutions to saddle paths can be obtained and so students can explicitly graph these solutions. Since it was the nature of saddle paths which gave students the greatest conceptual difficulty, this approach soon provided students with the insight into their nature that was lacking from a much more formal approach. Furthermore, they acquired this insight by explicitly dealing with an economic model.

I was much encouraged by the students' attitude to this 'problem oriented' approach. The course work assignments that I set were far too long and required far more preparation than could possibly be available under examination conditions. However, the students approached them with vigour during their course work period. Furthermore, it led to greater exchanges between students and a positive externality resulted.

This book is an attempt to bring this material together, to extend it, and make it more widely available. It is suitable for core courses in economic theory, and reading for students undertaking postgraduate courses and to researchers who require to acquaint themselves with the phase diagram technique. In addition, it can also be part of courses in quantitative economics. Outside of economics, it is also applicable to courses in mathematical modelling.

Finally, I would like to thank Cambridge University Press and the department of economics at Stirling for supplying the two mathematical software programmes; the copy editor, Anne Rix, for an excellent job on a complex manuscript; and my wife, Anne Thomson, for her tolerance in bringing this book about.

January 1997