CAMBRIDGE

Cambridge University Press
0521816580 - Differential Equations: Linear, Nonlinear, Ordinary, Partial
A. C. King, J. Billingham and S. R. Otto
Table of Contents
More information

Contents

Preface *page* ix

Part One: Linear Equations 1

1 Variable Coefficient, Second Order, Linear, Ordinary Differential Equations 3
1.1 The Method of Reduction of Order 5
1.2 The Method of Variation of Parameters 7
1.3 Solution by Power Series: The Method of Frobenius 11

2 Legendre Functions 31
2.1 Definition of the Legendre Polynomials, $P_n(x)$ 31
2.2 The Generating Function for $P_n(x)$ 35
2.3 Differential and Recurrence Relations Between Legendre Polynomials 38
2.4 Rodrigues' Formula 39
2.5 Orthogonality of the Legendre Polynomials 41
2.6 Physical Applications of the Legendre Polynomials 44
2.7 The Associated Legendre Equation 52

3 Bessel Functions 58
3.1 The Gamma Function and the Pockhammer Symbol 58
3.2 Series Solutions of Bessel's Equation 60
3.3 The Generating Function for $J_n(x)$, n an integer 64
3.4 Differential and Recurrence Relations Between Bessel Functions 69
3.5 Modified Bessel Functions 71
3.6 Orthogonality of the Bessel Functions 71
3.7 Inhomogeneous Terms in Bessel's Equation 77
3.8 Solutions Expressible as Bessel Functions 79
3.9 Physical Applications of the Bessel Functions 80

4 Boundary Value Problems, Green's Functions and Sturm–Liouville Theory 93
4.1 Inhomogeneous Linear Boundary Value Problems 96
4.2 The Solution of Boundary Value Problems by Eigenfunction Expansions 100
4.3 Sturm–Liouville Systems 107

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521816580 - Differential Equations: Linear, Nonlinear, Ordinary, Partial
A. C. King, J. Billingham and S. R. Otto
Table of Contents
More information

5 Fourier Series and the Fourier Transform 123
5.1 General Fourier Series 127
5.2 The Fourier Transform 133
5.3 Green's Functions Revisited 141
5.4 Solution of Laplace's Equation Using Fourier Transforms 143
5.5 Generalization to Higher Dimensions 145

6 Laplace Transforms 152
6.1 Definition and Examples 152
6.2 Properties of the Laplace Transform 154
6.3 The Solution of Ordinary Differential Equations using Laplace Transforms 157
6.4 The Inversion Formula for Laplace Transforms 162

7 Classification, Properties and Complex Variable Methods for Second Order Partial Differential Equations 175
7.1 Classification and Properties of Linear, Second Order Partial Differential Equations in Two Independent Variables 175
7.2 Complex Variable Methods for Solving Laplace's Equation 186

Part Two: Nonlinear Equations and Advanced Techniques 201

8 Existence, Uniqueness, Continuity and Comparison of Solutions of Ordinary Differential Equations 203
8.1 Local Existence of Solutions 204
8.2 Uniqueness of Solutions 210
8.3 Dependence of the Solution on the Initial Conditions 211
8.4 Comparison Theorems 212

9 Nonlinear Ordinary Differential Equations: Phase Plane Methods 217
9.1 Introduction: The Simple Pendulum 217
9.2 First Order Autonomous Nonlinear Ordinary Differential Equations 222
9.3 Second Order Autonomous Nonlinear Ordinary Differential Equations 224
9.4 Third Order Autonomous Nonlinear Ordinary Differential Equations 249

10 Group Theoretical Methods 256
10.1 Lie Groups 257
10.2 Invariants Under Group Action 261
10.3 The Extended Group 262
10.4 Integration of a First Order Equation with a Known Group Invariant 263

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521816580 - Differential Equations: Linear, Nonlinear, Ordinary, Partial
A. C. King, J. Billingham and S. R. Otto
Table of Contents
More information

10.5 Towards the Systematic Determination of Groups Under Which a First Order Equation is Invariant 265
10.6 Invariants for Second Order Differential Equations 266
10.7 Partial Differential Equations 270

11 Asymptotic Methods: Basic Ideas 274
11.1 Asymptotic Expansions 275
11.2 The Asymptotic Evaluation of Integrals 280

12 Asymptotic Methods: Differential Equations 303
12.1 An Instructive Analogy: Algebraic Equations 303
12.2 Ordinary Differential Equations 306
12.3 Partial Differential Equations 351

13 Stability, Instability and Bifurcations 372
13.1 Zero Eigenvalues and the Centre Manifold Theorem 372
13.2 Lyapunov's Theorems 381
13.3 Bifurcation Theory 388

14 Time-Optimal Control in the Phase Plane 417
14.1 Definitions 418
14.2 First Order Equations 418
14.3 Second Order Equations 422
14.4 Examples of Second Order Control Problems 426
14.5 Properties of the Controllable Set 429
14.6 The Controllability Matrix 433
14.7 The Time-Optimal Maximum Principle (TOMP) 436

15 An Introduction to Chaotic Systems 447
15.1 Three Simple Chaotic Systems 447
15.2 Mappings 452
15.3 The Poincaré Return Map 467
15.4 Homoclinic Tangles 472
15.5 Quantifying Chaos: Lyapunov Exponents and the Lyapunov Spectrum 484

***Appendix 1* Linear Algebra** 495

***Appendix 2* Continuity and Differentiability** 502

***Appendix 3* Power Series** 505

***Appendix 4* Sequences of Functions** 509

***Appendix 5* Ordinary Differential Equations** 511

***Appendix 6* Complex Variables** 517

***Appendix 7* A Short Introduction to MATLAB** 526

Bibliography 534
Index 536

© Cambridge University Press www.cambridge.org