# DYNAMICS OF ONE-DIMENSIONAL QUANTUM SYSTEMS

One-dimensional quantum systems show fascinating properties beyond the scope of the mean-field approximation. However, the complicated mathematics involved is a high barrier to non-specialists. Written for graduate students and researchers new to the field, this book is a self-contained account of how to derive a quasi-particle picture from the exact solution of models with inverse-square interparticle interactions.

The book provides readers with an intuitive understanding of exact dynamical properties in terms of exotic quasi-particles that are neither bosons nor fermions. Powerful concepts, such as the Yangian symmetry in the Sutherland model and its lattice versions, are explained. A self-contained account of non-symmetric and symmetric Jack polynomials is also given. Derivations of dynamics are made easier, and are more concise than in the original papers, so readers can learn the physics of one-dimensional quantum systems through the simplest model.

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# Dynamics of One-Dimensional Quantum Systems

## Inverse-Square Interaction Models

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## Preface

This book is concerned primarily with the exact dynamical properties of one-dimensional quantum systems. As a crucial property of exactly soluble models, we assume that the interaction decays as the inverse square of the distance. The family of these models is called the inverse-square interaction ($1/r^2$) models. In the one-dimensional continuum space, the model is often referred to as the Calogero–Sutherland model. In the one-dimensional lattice, on the other hand, the first $1/r^2$ models appeared as a spin model, which is now called the Haldane–Shastry model. Soon after the discovery of the Haldane–Shastry model, it was recognized that the imposition of supersymmetry allows the model to acquire the charge degrees of freedom, while keeping the exactly soluble nature. The resultant one-dimensional electron model is called the supersymmetric $t$–$J$ model. Various generalizations of these models have been proposed.

Recent experimental progress in quasi-one-dimensional electron systems, especially by neutron scattering and photoemission spectroscopy, has enhanced the theoretical motivation for exploring the dynamics over a wide frequency and momentum range. The $1/r^2$ models are ideally suited to meet this situation, since the model allows derivation of exact dynamical information most easily and transparently. In spite of the special appearance of the $1/r^2$ models, the intuition thus obtained contributes greatly to understanding low-dimensional physics in general. This kind of approach to dynamics is complementary to another powerful approach using the bosonization and conformal field theory. The latter is especially suitable to asymptotics of correlation functions at long spatial and temporal distances.

The literature relevant to the $1/r^2$ models is vast and scattered. Moreover, many papers include a difficult-looking mathematical set-up. This situation may cause newcomers to see a barrier too high to jump over before enjoying the rich and beautiful ingredients of the $1/r^2$ models. For several years, the authors have realized the necessity of a comprehensive treatise. This book is intended to be accessible to non-specialists who are interested in strongly correlated quantum systems. It explains the wonderfully beautiful physics and related mathematics in a self-contained manner, without assuming special knowledge on theories in one dimension. In order to make a coherent discussion, we have included many results that are newly derived for this book, in addition to summarizing what has been reported in the literature. We hope that this book is useful not only to experts already working in the field, but also to graduate students and researchers trying to delve into the fascinating physics in low dimensions.

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