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Excerpt  
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## PART ONE

# GENERAL CONCEPTS

# 1 Flow properties and basic principles

Before being able to measure a flow property, it is necessary to understand its nature and its relationship to other properties. Furthermore, for a proper use of a measuring instrument, one must be thoroughly familiar with the principles of its operation, which usually involve concepts and relationships from several different fields. In this chapter, the basic principles of fluid mechanics are reviewed and the properties of interest and their groupings in dimensionless form are identified. This is meant to be a refresher of familiar concepts, as well as to identify a possible need for more in-depth reviews of fluid mechanics [1–3], thermodynamics [4], and heat transfer [5]. Background material from system dynamics, signal analysis, and optics is reviewed separately in later chapters.

## 1.1 Forces, stresses, and the continuum hypothesis

All material objects are subjected to external forces, which are of two types, body forces and surface forces. *Body forces* act on the bulk of the object from a distance and are proportional to its mass; the most common examples are gravitational and electromagnetic forces. *Surface forces* are exerted on the surface of the object by other objects in contact with it; they generally increase with increasing contact area.

Any surface force acting on an elementary surface section of an object can be decomposed into a *normal* component, with a direction normal to the local tangent plane, and a *tangential* or *shear* component, with a direction parallel to the local tangent plane (see Fig. 1.1).

The *stress* at a point of an object is defined as the corresponding surface force per unit area; consequently there are two types of stresses, *normal stresses* and *shear stresses*. With respect to a Cartesian coordinate system, all stresses acting on three planes normal to the three axes form a second-order Cartesian tensor, which has nine components, only six of which (three normal stresses and three shear stresses) are independent. In classical fluid mechanics, the (static) *pressure* is defined as the average normal stress along any three orthogonal directions.

According to the classical definition, a *fluid* is a material that cannot withstand a shear stress when at rest; in other words, a fluid subjected to a shear stress will always be in motion or deformation. Fluids are easily deformable materials and take the shape of

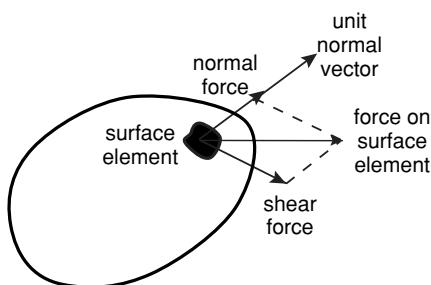


Figure 1.1. Force on a surface element and its decomposition to normal and shear components.

any container in which they are contained. They are further distinguished into *liquids*, which have a relatively high density and require an extremely large change of normal stresses for a change of their volume, and *gases*, which have a relatively low density and can easily change their volume. Unlike liquids, gases tend to occupy the entire available volume of their container. Besides ‘simple’ liquids and gases, there are a number of materials that, although not satisfying all properties of classical fluids, exhibit fluid-like properties. Examples include viscoelastic materials, which may sustain a certain amount of shear stress without being set in motion but behave like fluids when the shear stress exceeds a certain level, and plasmas, which form when gases are exposed to extremely high temperatures, in which case their molecules are dissociated into free atoms.

In most applications within the scope of conventional fluid mechanics, the phenomena of interest are characterized by scales that are far larger than the distances between molecules. Then the flow properties are defined as statistical averages over a volume that contains a very large number of molecules. In such cases, one need not be concerned about individual molecular or atomic motions and masses; instead, one should invoke the *continuum hypothesis*, by which any property of the fluid is assumed to have a continuous distribution within the volume of the fluid. Thus one may define the *local* value of the property as the limit of the volume-averaged value of this property as the volume collapses towards a mathematical point. One may refer to a *fluid element*, or *fluid particle*, as a material entity that has an infinitesimal volume, in which case its properties are uniform within this volume. In multiphase flows of immiscible fluids, the continuum hypothesis applies within each individual fluid, whereas some properties may be considered as discontinuous at the interface. Obviously there are also situations in which the continuum hypothesis does not apply at all; for example, in rarefied gases, in which the distances between gas molecules are relatively large, one must account for individual molecules and their motions.

## 1.2 Measurable properties

A property of a fluid element can be measured directly or estimated from measurements of other properties only if it has a precise and unambiguous scientific definition, associated with a measurement procedure. The following list identifies measurable properties of common interest, classified into four general classes.

**1.3 FLOW VELOCITY AND VELOCITY FIELDS****5**

- *Material properties:*

mass	density	specific volume
viscosity	thermal conductivity	molecular diffusivity
specific heat under constant pressure	specific heat under constant volume	gas constant
bulk modulus of elasticity	coefficient of thermal expansion	electric conductivity
surface tension	index of refraction	fluorescence

- *Kinematic properties*, namely properties that describe the motion of a fluid without consideration of applied forces:

position	displacement	velocity
volume flow rate	mass flow rate	acceleration
vorticity	strain rate	angular position
angular displacement	angular velocity	angular acceleration
momentum	angular momentum	

- *Dynamic properties*, namely properties related to the applied forces:

force	stress	torque
pressure (mechanical definition)		

- *Thermodynamic properties*, namely properties related to heat and work:

temperature	internal energy	enthalpy
entropy	heat flux	work
energy	pressure (thermodynamic definition)	

Material properties are usually not the subject of experimental fluid mechanics, as their values can be found in handbooks [6] or other sources. However, if a material property is unknown or overly sensitive to the particular experimental conditions, its value may have to be determined either as part of the overall experiment or by a specific experimental investigation.

**1.3 Flow velocity and velocity fields**

A position in space is specified in terms of its coordinates  $x_i$ ,  $i = 1, 2, 3$ , with respect to a Cartesian coordinate system. At any time  $t$ , this position is occupied by some fluid element, assumed to maintain its mass within an infinitesimally small volume. With the fluid considered as a continuum, we may define the flow velocity at a given position and a given time as the velocity of a fluid element that occupies that position at that time. We are also interested in defining the *velocity field*, which consists of the velocities of all fluid elements that comprise a material system. Thus it becomes necessary to distinguish the fluid element in question from any other fluid element. For clarity, we specify as  $X_i$  the coordinates of the fluid element that occupies position  $x_i$  at time  $t$ . This element moves along its trajectory, indicated by the dotted curve in Fig. 1.2. At time  $t + \delta t$ , the

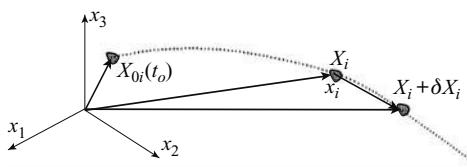


Figure 1.2. Fluid element positions.

same element will have the coordinates  $X_i + \delta X_i$ . Then the *flow velocity* is defined as

$$V_i = \lim_{\delta t \rightarrow 0} \frac{\delta X_i}{\delta t} = \frac{dX_i}{dt}. \quad (1.1)$$

One approach to identifying the fluid element is to specify its initial coordinates  $X_{0i}$ , namely the coordinates of the position that the element occupied at the origin of time  $t_o$ . Then its coordinates at any time  $t$  are functions of only the initial coordinates and  $t$ , namely  $X_i = X_i(X_{0i}, t)$ . The velocity field may also be specified as a function of these two variables, as  $V_i = V_i(X_{0i}, t)$ . This approach is known as the *material* or *Lagrangian* description of flow motion. Because identifying ('tagging') individual fluid elements is not usually practical, it is also customary to express the velocity field in terms of a fixed position with respect to the coordinate system and, of course, time; this approach is known as the *spatial* or *Eulerian* description. To avoid confusion when differentiating, the velocity field according to the Eulerian description is denoted by a different symbol, as  $U_i(x_i, t)$ ; it is understood nevertheless that, at all positions and for all times, the definition of flow velocity is unique and that

$$V_i(X_{0i}, t) = U_i(x_i, t). \quad (1.2)$$

Following the Lagrangian description, the fluid element *acceleration* is defined as

$$a_i = \frac{dV_i}{dt}. \quad (1.3)$$

However, if we follow the Eulerian description, we must account for changes of the velocity from one location of the particle to the next, and the fluid acceleration becomes

$$a_i = \frac{\partial U_i}{\partial t} + U_1 \frac{\partial U_i}{\partial x_1} + U_2 \frac{\partial U_i}{\partial x_2} + U_3 \frac{\partial U_i}{\partial x_3}, \quad (1.4)$$

where the first term on the right-hand side is called the *local acceleration* and the remaining terms are called the *convective acceleration*. The right-hand side of Eq. (1.4) is called the *material* or *substantial derivative* of  $U_i$  and is usually denoted as  $DU_i/Dt$ .

Fluid deformation under the influence of stresses is described by the *rate of strain*, or *rate of deformation, tensor*:

$$e_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \quad i, j = 1, 2, 3. \quad (1.5)$$

## 1.4 ANALYTICAL DESCRIPTION OF FLOWS

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**1.4 Analytical description of flows**

The physical relationships among various flow properties are represented by analytical expressions, which constitute algebraic, differential, integral, or integrodifferential equations. Such relationships include *axiomatic principles*, which are generally accepted as natural laws subjected to experimental verification, and *empirical relationships*, which range from semi-theoretical concepts, based on physical arguments with some experimental input, to purely empirical expressions, obtained by statistical curve fitting to experimental results.

Conventional fluid mechanics and thermodynamics are based on four basic axiomatic principles: the conservation of mass, the momentum principle, and the first and second laws of thermodynamics. When applied to a *closed fluid system*, these principles result in a set of integral relationships that describe the fluid motion. However, it is usually more convenient to apply these principles to the contents of a fixed *control volume*, in which case one may derive the following four basic relationships.

- The *conservation of mass* or *continuity* equation:

$$\frac{\partial}{\partial t} \int_V \rho dV + \int_A \rho \vec{V} \cdot d\vec{A} = 0, \quad (1.6)$$

where  $\rho$  is the fluid density,  $\vec{V}$  is the velocity vector,  $V$  is the control volume, and  $A$  is the area of its surface. For non-reacting multicomponent flows, one may formulate similar equations expressing the *conservation of species*.

- The *momentum* equation (Newton's second law), which, for an inertial coordinate system, can be written as

$$\vec{F} = \frac{\partial}{\partial t} \int_V \vec{V} \rho dV + \int_A \vec{V} \rho \vec{V} \cdot d\vec{A}, \quad (1.7)$$

where  $\vec{F}$  is the net external force acting on the control volume.

- The *energy* equation (first law of thermodynamics):

$$\dot{Q} - \dot{W} = \frac{\partial}{\partial t} \int_V \left( u + \frac{1}{2} V^2 + gz \right) \rho dV + \int_A \left( u + \frac{p}{\rho} + \frac{1}{2} V^2 + gz \right) \rho \vec{V} \cdot d\vec{A}, \quad (1.8)$$

where  $\dot{Q}$  is the rate of heat transfer from the surroundings to the control volume,  $\dot{W}$  is the mechanical power produced by moving solid components, shear stresses acting on the boundary or electromagnetic forces, but not normal stresses acting on the control volume (the latter have been included on the right-hand side),  $u$  is the specific internal energy,  $p$  is the pressure,  $z$  is an upwards vertical axis, and  $g$  is the gravitational acceleration.

- The *second law* of thermodynamics:

$$\frac{\partial}{\partial t} \int_V s \rho dV + \int_A s \rho \vec{V} \cdot d\vec{A} \geq \int_V \frac{1}{T} \left( \frac{\dot{Q}}{A} \right) dA, \quad (1.9)$$

where  $s$  is the specific entropy and  $T$  is the absolute temperature.

By letting the control volume vanish towards a point, one may convert the previous equations into differential forms (see examples in the next section). These must be complemented by initial and boundary conditions. The usual conditions employed at a solid boundary in conventional fluid mechanics are the *no-penetration* condition, which specifies that the relative velocity normal to the contact surface vanishes, and the *no-slip* condition, which specifies that the relative velocity tangential to the contact surface vanishes. In two-phase flows, the pressure difference  $\Delta p$  across the interface is related to the surface tension  $\sigma$  by the expression

$$\Delta p = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right), \quad (1.10)$$

where  $R_1$  and  $R_2$  are the principal radii of curvature; obviously,  $\Delta p = 0$  across plane interfaces. In liquid–gas flows, it is customary to neglect the shear stress on the liquid side of the interface (*stress-free boundary*).

Certain types of flow require the use of additional axiomatic principles, such as the following ones:

- The *laws of chemical reaction* for reactive flows.
- *Magnetohydrodynamic laws* for electrically conductive fluids in magnetic fields.

Among the common types of empirical relationships, one may mention the following ones:

- *Equations of state or constitutive relationships*, relating various thermodynamic properties of a material. The simplest example of an equation of state is the perfect-gas law,

$$\frac{P}{\rho} = RT \quad (1.11)$$

where  $R$  is a gas constant.

- *Stress–strain rate* (also called ‘constitutive’) *relationships*. Besides the linear stress–strain rate relationship that describes *Newtonian fluids*, a variety of other such relationships have been proposed to describe deformation of *non-Newtonian fluids*.
- *Turbulence models*, which are relationships among various statistical properties of turbulent flows. A widely used turbulence model is the gradient transport model, by which the turbulent stresses are linearly related to the mean strain.

## 1.5 The choice of analytical approach

Although accurate mathematical models of fluid flow are available, it is generally advisable, and even necessary, to employ simplifications whenever possible and to the greatest possible extent. This strategy has been applied extensively to the analysis of measuring instrument operation. As a rule, one should strive to use the simplest possible analytical model that permits the desired measurement with an acceptable uncertainty. Of course, the differences between an approximate and a more ‘exact’ method must be analysed,

## 1.5 THE CHOICE OF ANALYTICAL APPROACH

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and, if they are found to be excessive, one must either apply appropriate corrections or abandon the approximate method in favour of a more exact one.

Among the important effects, which may or may not be accounted for in a particular type of analysis, are deformation, friction, compressibility, and turbulence; additional effects complicating the analysis may be present under specific circumstances. Accordingly, one may distinguish the following theoretical approaches used commonly to describe fluid motion:

- Fluid statics.** When a fluid is at rest, or in rigid-body motion, it does not deform, and therefore it cannot sustain shear stresses. In fluid statics, the three normal stresses are equal in magnitude to each other and to the pressure. Static fluids are subjected to gravity, which causes the development of hydrostatic pressure and buoyancy force. Static fluid analysis is useful in analyzing the performance of many instruments, notably those of manometers, barometers, and certain types of pressure transducers. On the other hand, the use of static fluid analysis in situations in which there are fluids in motion could lead to substantial errors. For example, a static analysis of a liquid manometer subjected to pressure fluctuations would result in erroneously low readings of pressure differences.
- Inviscid incompressible flows.** The simplest mathematical model of fluid flow is one that neglects the effects of friction and compressibility. Continuity imposes the requirement of conservation of volume of an incompressible fluid. In differential form, this can be expressed as

$$\frac{\partial U_1}{\partial x_1} + \frac{\partial U_2}{\partial x_2} + \frac{\partial U_3}{\partial x_3} = 0. \quad (1.12)$$

When friction is neglected, turbulence must also be disregarded. The differential momentum equation for an *inviscid, incompressible* fluid is known as the *Euler* equation:

$$\frac{DU_i}{Dt} = g_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i}. \quad (1.13)$$

Together, the continuity equation and the Euler equation form a closed system, which is sufficient for the determination of the fluid velocity and pressure. Integration of Euler's equation along a streamline leads to the simple and frequently used (as well as misused), steady-flow *Bernoulli's* equation, which is an algebraic expression relating velocity magnitude and pressure as

$$p + \frac{1}{2} \rho U^2 + \rho g z = \text{const.} \quad (1.14)$$

The flow analysis can be further simplified by the assumption of *irrotationality*, namely the vanishing of *vorticity*,

$$\vec{\zeta} = \text{curl } \vec{U}, \quad (1.15)$$

everywhere in the flow domain, with the possible exception of isolated singularities. Such a flow is called *potential* and is described by the *velocity potential*, which

satisfies Laplace's equation. In potential flow, Bernoulli's equation can be applied not only along a streamline but also from one streamline to another. Potential flow analysis is a common approach in aerodynamics. It is also used to explain the operation of several simple instruments. For example, the measurement of velocity with the use of immersed pressure tubes routinely employs the steady-flow Bernoulli's equation; this approach is acceptable in many wind-tunnel applications, in which the effects of friction are known to be below the uncertainty level, but it introduces large errors in measurements near walls, where friction effects are important, or in high-speed flows, for which compressibility must be accounted.

3. **Viscous incompressible flows.** Application of Newton's second law to an incompressible, Newtonian fluid with constant material properties leads to the *Navier–Stokes* equations:

$$\frac{DU_i}{Dt} = g_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \left( \frac{\partial^2 U_i}{\partial x_1^2} + \frac{\partial^2 U_i}{\partial x_2^2} + \frac{\partial^2 U_i}{\partial x_3^2} \right), \quad (1.16)$$

where  $\nu$  is the kinematic viscosity. The main parameter characterizing such flows is the *Reynolds number*  $Re$ , which represents the ratio of inertia forces and viscous forces. In the limiting case of vanishing  $Re$ , inertial effects become negligible and the Navier–Stokes equation is reduced to the *Stokes* equation [3, 7], which contains no non-linear terms. The form of applicable solutions and the measuring instrument response in low- $Re$  flows are radically different from those at higher  $Re$ . On the other extreme, as  $Re \rightarrow \infty$ , one would expect that viscous effects should become negligible. However, this limit contradicts the physical fact that, as the Reynolds number increases, a flow would become increasingly unstable and eventually turbulent. As a rule, at high  $Re$ , one must consider friction as well as account for turbulence effects. The infinite- $Re$  limit also leads to an analytical singularity because the order of the Navier–Stokes equation changes from second to first.

4. **Compressible flows.** In compressible flow, density variation is significant. Density changes are related to pressure changes and are also accompanied by temperature changes. Unlike incompressible flow, compressible flow is not divergence free, and the corresponding differential continuity equation includes the unknown variable density. Compressible flow models contain, besides velocity and pressure, two additional unknowns, density and temperature, and therefore require two additional equations. One independent differential equation (*energy equation*) is provided by the first law of thermodynamics, and a second relationship is provided by an equation of state. To simplify the analysis of compressible flow, friction is often neglected. *Isentropic* flow, namely a reversible and adiabatic change of state, is commonly used as an approximation in compressible flow instrumentation. Allowing for density changes permits the modelling of propagation of weak disturbances with a finite speed, which, for isentropic flows, is called the *speed of sound*. The ratio  $M$  of fluid velocity to the speed of sound, called the *Mach number*, is a dimensionless parameter describing the effects of compressibility. It can be viewed as representing the ratio of inertia forces to elastic forces. When  $M > 1$  (supersonic flow), dramatic and

sudden changes of fluid velocity and thermodynamic properties may occur across a *shock wave*, which is an irreversible, and therefore non-isentropic, process.

5. **Turbulent flows.** Turbulence is a state of motion in which flow properties, including velocity, pressure, and vorticity, vary rapidly and randomly in space and time. Randomness requires a statistical description, which is customarily obtained by the decomposition of each variable property into a *mean* and a *fluctuation* (*Reynolds decomposition*). Statistical equations for the means and fluctuations have been formulated but lead to systems containing unknowns whose number exceeds the number of available equations. Turbulence affects the operation of even the simplest instruments. In some cases, relatively simple corrections for turbulence effects have been devised. An important parameter is the *turbulence intensity*, i.e., the ratio of the root-mean-square velocity fluctuation and the corresponding mean velocity; however, information on the time and length scales of turbulent motions is often required in the correction method. The measurement of turbulent properties themselves can be achieved with the use of special instrumentation that has particularly refined spatial, temporal, and amplitude resolutions. Special methods, including *phase averaging* and *conditional sampling*, have been developed to resolve quasi-deterministic turbulence patterns, known as *coherent structures*.
6. **'Complex' flows.** Fluids and flows are sometimes complicated by particular conditions that necessitate the use of specialized analytical models and methods as well as special instrumentation, calibration, and measuring procedures. Of particular interest are the following cases:
  - *Multiphase* flows, including liquid–solid, gas–solid, gas–liquid, and liquid–liquid flows, and flows with phase change.
  - Flows with *chemical reaction*, including combustion. These involve changes of composition, accompanied by release or absorption of chemical energy and changes of temperature.
  - *Low-density (rarefied)* gas flows, for which the continuum hypothesis is inappropriate and whose measurement relies on molecular and atomic phenomena.
  - Flows of *non-Newtonian* fluids having nonlinear stress–strain rate relationships (e.g., polymer flows).
  - *Magnetohydrodynamic* flows, namely flows of electrically conductive fluids in magnetic fields.

## 1.6 Similarity

**Similarity and non-dimensionalization:** Measurable properties may be either *primary*, namely independent of each other, or *secondary*, namely related to other properties through their definition or a basic principle. International conventions regulate the choice of a sufficient set of primary properties (called *dimensions*) and appropriate standard amounts (called *units*) that may be used as scales for expressing magnitudes. The SI (Système International d'Unités) System of Units has adopted the following properties