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Giri Narasimhan, Michiel Smid
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Geometric Spanner Networks

Aimed at an audience of researchers and graduate students in computational geometry and algorithm design, this book uses the Geometric Spanner Network Problem to showcase a number of useful algorithmic techniques, data structure strategies, and geometric analysis techniques with many applications, practical and theoretical.

The authors present rigorous descriptions of the main algorithms and their analyses for different variations of the Geometric Spanner Network Problem. Although the basic ideas behind most of these algorithms are intuitive, very few are easy to describe and analyze. For most of the algorithms, nontrivial data structures need to be designed, and nontrivial techniques need to be developed in order for analysis to take place. Still, there are several basic principles and results that are used throughout the book. One of the most important is the powerful well-separated pair decomposition. This decomposition is used as a starting point for several of the spanner constructions.

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To my parents, Kalyani and Narasimhan, who taught me the fundamentals of life and the pursuit of knowledge and excellence, and to Kalai Mathee, with whom I collaborate on the advanced concepts. (G.N.)

To my parents, Nell and Giel, who (unknowingly) convinced me to become a mathematician and theoretical computer scientist. (M.S.)

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Preface

> Philosophy is written in this grand book – I mean universe – which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it.
>
> —Galileo, *The Assayer (Il saggiatore)*, 1623

This book started as a collection of personal notes on Geometric Spanner Networks. Over time, these notes grew, and we realized that they could be of value to many researchers in the field. Gautam Das suggested that it be turned into a monograph. It made sense, because the geometric spanner problem is closely related to several fundamental problems in geometric and graph algorithms, including the minimum spanning tree problem, the Steiner minimum tree problem, the traveling salesperson problem, and the shortest path problem. We assume that the reader has had previous exposure to (undergraduate-level) basic concepts of discrete mathematics, data structures, probability and combinatorics, algorithm analysis, fundamental algorithms, and mathematical proof techniques. This book can serve as a reference text and can also be used as a self-study book for anyone interested in research in computational geometry and geometric algorithms.

One of the main features of this book is its attention to detail – detail in the proofs and arguments presented. We have striven to present complete proofs, wherever possible and appropriate, while at the same time peppering it with intuition, so that the reader can understand the underlying train of thought.

While there are numerous examples of the design of efficient algorithms much before that, by 1864 Charles Babbage foresaw rather clearly the development of the field of algorithms when he wrote in *The Life of a Philospher*:

> As soon as an *Analytical Engine* exists, it will necessarily guide the future course of the science. Whenever any result is sought by its aid, the question will then arise – by what course of calculation can these results be arrived at by the machine in the shortest time?

Many of us were drawn to the field of algorithms because of the elegance, subtlety, precision, and clarity of the ideas and arguments. This is especially true of some of the early work in the field, some of which is now part of undergraduate texts. The field has evolved since then. The algorithms and data structures and analysis tools have become more complex and sophisticated. To do research and to keep up with the advances in the field, it is necessary to know how to read papers, focus on their central ideas, and skip unnecessary details that may cloud an elegant argument. But this requires training and skill, and it is particularly challenging for the novice, the beginning graduate student. As

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academics, we already know that "teaching is not the mere imparting of information but the cultivation of an inquiring mind" (J. Krishnamurthy, *Life Ahead* (1963)). But we face a greater challenge – one of capturing and maintaining students' interest and keeping them challenged. Details, when presented in papers, are often boring and time-consuming, especially when the underlying ideas are new. Details are hard to write down. So, it is with good reason that details are often pushed "under the rug."

Then why should the novice be concerned about understanding details? It takes skill and training to convert good ideas and great intuition into an end result (an algorithm, a proof, etc.). When writing a paper, without the attention to details, one is more prone to errors. It is easier to make wild claims and statements without details, and it is harder to see subtle errors without the attention to details. Many of us have refereed papers for journals and conferences, where this is certainly a critical issue. Also, details help you "take apart" an algorithm or a proof; details help you understand limitations that are not obvious from an overview; details will help you understand the hurdles that must be overcome to make improvements; finally details will help you innovate and dig deeper.

Web site

This book has its own Web site(s); the following are mirror sites:

`www.scs.carleton.ca/~michiel/SpannerBook.html`
`www.cis.fiu.edu/~giri/SpannerBook.html`

They will contain a list of known errors and new developments in Geometric Spanner Networks.

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Parts of this book were used as lecture notes at the Max-Planck-Institute for Computer Science in Saarbrücken and for courses at the University of Magdeburg and Carleton University. We thank the students in Magdeburg and Ottawa for their enthusiasm and feedback.

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