

Contents

Preface	<i>page</i> xiii
1 Classical Regular Polytopes	1
1A The Historical Background	1
1B Regular Convex Polytopes	7
1C Extensions of Regularity	15
1D Regular Maps	17
2 Regular Polytopes	21
2A Abstract Polytopes	22
2B Regular Polytopes	31
2C Order Complexes	39
2D Quotients	42
2E C-Groups	49
2F Presentations of Polytopes	60
3 Coxeter Groups	64
3A The Canonical Representation	64
3B Groups of Spherical or Euclidean Type	71
3C Groups of Hyperbolic Type	76
3D The Universal Polytopes $\{p_1, \dots, p_{n-1}\}$	78
3E The Order of a Finite Coxeter Group	83
4 Amalgamation	95
4A Amalgamation of Polytopes	96
4B The Classification Problem	101
4C Finite Quotients of Universal Polytopes	103
4D Free Extensions of Regular Polytopes	106
4E Flat Polytopes and the FAP	109
4F Flat Polytopes and Amalgamation	115

5 Realizations	121
5A Realizations in General	121
5B The Finite Case	127
5C Apeirotopes	140
6 Regular Polytopes on Space-Forms	148
6A Space-Forms	148
6B Locally Spherical Polytopes	152
6C Projective Regular Polytopes	162
6D The Cubic Toroids	165
6E The Other Toroids	170
6F Relationships Among Toroids	172
6G Other Euclidean Space-Forms	175
6H Chiral Toroids	177
6J Hyperbolic Space-Forms	178
7 Mixing	183
7A General Mixing	183
7B Operations on Regular Polyhedra	192
7C Cuts	201
7D The Classical Star-Polytopes	206
7E Three-Dimensional Polyhedra	217
7F Three-Dimensional 4-Apeirotopes	236
8 Twisting	244
8A Twisting Operations	244
8B The Polytopes $\mathcal{L}^{\mathcal{K},\mathcal{G}}$	247
8C The Polytopes $2^{\mathcal{K}}$ and $2^{\mathcal{K},\mathcal{G}(s)}$	255
8D Realizations of $2^{\mathcal{K}}$ and $2^{\mathcal{K},\mathcal{G}(s)}$	259
8E A Universality Property of $\mathcal{L}^{\mathcal{K},\mathcal{G}}$	264
8F Polytopes with Small Faces	272
9 Unitary Groups and Hermitian Forms	289
9A Unitary Reflexion Groups	290
9B Hermitian Forms and Reflexions	298
9C General Considerations	305
9D Generalized Triangle Groups	320
9E Tetrahedral Diagrams	332
9F Circuit Diagrams with Tails	347
9G Abstract Groups and Diagrams	355
10 Locally Toroidal 4-Polytopes: I	360
10A Grünbaum's Problem	360
10B The Type $\{4,4,3\}$	363
10C The Type $\{4,4,4\}$	369
10D Cuts for the Types $\{4, 4, r\}$	378
10E Relationships Among Polytopes of Type $\{4, 4, r\}$	383

11 Locally Toroidal 4-Polytopes: II	387
11A The Basic Enumeration Technique	387
11B The Polytopes ${}_p\mathcal{T}_{(s,0)}^4 := \{\{6, 3\}_{(s,0)}, \{3, p\}\}$	392
11C Polytopes with Facets $\{6, 3\}_{(s,s)}$	400
11D The Polytopes ${}_6\mathcal{T}_{(s,0),(t,0)}^4 := \{\{6, 3\}_{(s,0)}, \{3, 6\}_{(t,0)}\}$	410
11E The Type $\{3, 6, 3\}$	417
11F Cuts of Polytopes of Type $\{6, 3, p\}$ or $\{3, 6, 3\}$	423
11G Hyperbolic Honeycombs in \mathbb{H}^3	431
11H Relationships Among Polytopes of Types $\{6, 3, p\}$ or $\{3, 6, 3\}$	437
12 Higher Toroidal Polytopes	445
12A Hyperbolic Honeycombs in \mathbb{H}^4 and \mathbb{H}^5	445
12B Polytopes of Rank 5	450
12C Polytopes of Rank 6: Type $\{3, 3, 3, 4, 3\}$	459
12D Polytopes of Rank 6: Type $\{3, 3, 4, 3, 3\}$	462
12E Polytopes of Rank 6: Type $\{3, 4, 3, 3, 4\}$	465
13 Regular Polytopes Related to Linear Groups	471
13A Regular Polyhedra	471
13B Connexions Among the Polyhedra	478
13C Realizations of the Polyhedra	484
13D The 4-Polytopes	490
13E Connexions Among 4-Polytopes	500
14 Miscellaneous Classes of Regular Polytopes	502
14A Locally Projective Regular Polytopes	502
14B Mixed Topological Types	509
Bibliography	519
Indices	
List of Symbols	539
Author Index	543
Subject Index	544