

Fluid Dynamics of Particles, Drops, and Bubbles

This book is a modern presentation of multiphase flow, from basic principles to state-of-the-art research. It explains dispersed fluid dynamics for bubbles, drops, or solid particles, incorporating detailed theory, experiments, simulations, and models while considering applications and recent cutting-edge advances.

The book demonstrates the importance of multiphase flow in engineering and natural systems, considering particle size distributions, shapes, and trajectories as well as deformation of fluid particles and multiphase flow numerical methods. The scope of the book also includes coupling physics between particles and turbulence through dispersion and modulation, and specific phenomena such as gravitational settling and collisions for solid particles, drops, and bubbles. The eight course-based chapters feature over 100 homework problems, including theory-based and engineering application questions. The final three reference-based chapters provide a wide variety of particle point-force theories and models.

The comprehensive coverage will give the reader a solid grounding for multiphase flow research and design, applicable to current and future engineering. This is an ideal resource for graduate students, researchers, and professionals.

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Preface

This book provides an overview of dispersed multiphase fluid dynamics of bubbles, drops, and solid particles, whose trajectory is primarily determined by the surrounding flow. The motion of the particles and impact on the fluid is described using governing equations, basic physics, theory, experimental/numerical results, and empirical models.

The text is organized in 11 chapters that provide an introduction, basic equations of motion, particle and coupling classifications, particle–turbulent interactions, an overview of multiphase numerical methods, and reference details for point-force approaches.

The first eight chapters are designed for a course in multiphase flow. The introduction of Chapter 1 describes the importance of multiphase flow to engineering and natural systems. Chapter 2 is a brief review of single-phase fluid dynamic equations, which are used in Chapter 3 to obtain equations of motion for a single spherical particle. Chapter 4 considers particle size distributions, shapes, and trajectories as well as deformation of fluid particles (drops and bubbles). Chapter 5 characterizes multiphase flow-coupling interactions based on particle concentration, along with coupling regime classification. Chapter 6 reviews basic principles of single-phase turbulence, which are used in Chapter 7 to identify coupling physics between particles and turbulence. Chapter 8 overviews numerical methods for multiphase flows and discusses the pros and cons of various approaches. Chapters 1–8 are written for use in a course and have recommended homework problems.

Reference details on the point-force model are provided in Chapters 9–11. Chapter 9 focuses on the fluid dynamic drag on a particle for a wide variety of conditions, while Chapter 10 considers details of lift and other fluid dynamic forces for one-way coupling. Three-way and four-way coupling effects for a point-force description are reviewed in Chapter 11.

Additional information for multiphase flow physics is available in the books of Wallis (1969), Clift et al. (1978), Soo (1990), Drew and Passman (1998), Kleinstreuer (2003), Brennen (2005), Michaelides (2006), Leal (2007), Prosperetti and Tryggvason (2007), Crowe et al. (2011), and Marshall and Li (2014). Heat and mass transfer aspects are also discussed by Williams (1965), Kuo (1986), Oran and Boris (1987), and Sirignano (2010), while dense flow treatments (where particle collisions dominate) are covered by Gidaspow (1994) and Marchisio and Fox (2013).

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This book endeavors to present dispersed multiphase fluid dynamics within a simple engineering discussion, based on research in the community by my colleagues, whom I thank. I am indebted to Professor G. M. Faeth (1936–2005), who introduced me to the subject, the many great students at the University of Illinois and the University of Virginia who provided valuable feedback (you know who you are!), as well as the team at Cambridge University Press for their superb editorial guidance.

I dedicate this book to my family for all their support, especially my wife Marie.

To honor these people, 100% of the author's proceeds from this book will be donated to the International Committee of the Red Cross (ICRC). The opinions expressed in this text are those of the author (and not necessarily of the ICRC).

Nomenclature

Unless otherwise stated, the numbers in parentheses correspond to the numbered equations presented throughout the text.

Roman Letters

<i>a</i>	Spheroid angle of attack (10.42)
<i>a</i>	Speed of sound (2.42)
<i>A</i>	Area or boundary surface (1.2)
<i>A</i>	Bubble oscillation amplitude (4.59)
<i>A*</i>	Normalized surface area (9.49)
<i>b</i>	Richardson–Zaki exponent (11.17)
<i>B</i>	Spalding transfer number (3.100)
<i>B</i>	Compressibility pressure constant (2.36)
<i>Bo</i>	Bond number (4.43)
<i>c</i>	Coefficient, such as (3.76)
<i>c</i>	Specific heat (2.29)
const.	Arbitrary constant of order unity
<i>C</i>	Force coefficient, such as (2.87)
<i>C</i>	Cumulative distribution function (4.4)
<i>Ca</i>	Capillary number (4.61a)
<i>d</i>	Equivalent volumetric diameter of a particle (1.1b)
<i>d</i>	Particle-path derivative (1.17a) or diameter derivative (4.3b)
<i>D</i>	Macroscopic length scale of physical domain (1.3)
<i>D</i>	Fluid path derivative (1.17b)
<i>e</i>	Internal energy (2.23)
<i>e</i>	Coefficient of restitution (8.32a)
<i>E</i>	Particle aspect ratio (4.38)
<i>E</i>	Turbulent kinetic energy per wavenumber (6.93)
<i>E</i>	Young's modulus (11.39)
<i>f</i>	Stokes drag correction factor (3.31)
<i>f</i>	Ordinary frequency (2.95)
<i>F</i>	Turbulent kinetic energy per frequency (6.97)
<i>F</i>	Marker function (8.53)
F	Force (3.2)

Fr	Froude number (2.71)
\mathbf{g}	Gravity acceleration vector (2.13)
g	Gravity drift parameter (7.18)
\mathcal{G}	Flow strain (4.67a)
\mathbf{G}	Gradient of viscous stresses (2.11)
h	Spread parameter for a size distribution (4.24)
\hat{h}	Enthalpy (2.38b)
H	History force kernel (10.53a)
\mathbf{i}	Unit vector (1.8)
\mathbf{I}	Impulse (11.32a)
I	Moment of inertia (4.40)
I^*	Normalized inertia about broadside axis (4.40)
J^*	Normalized McLaughlin lift parameter (10.9)
k	Turbulent kinetic energy of the surrounding fluid (6.17)
κ	Thermal conductivity (2.25)
\mathcal{K}	Compressibility exponent (2.36)
K_{ij}	Viscous stress tensor on face i in direction j (2.15a)
Kn	Knudsen number (1.20)
l	Distance (1.20) or wavelength (2.91)
m	Mass (1.4)
u	Viscosity ratio function (9.68b)
m	Particle mass loading (5.6)
\dot{m}	Mass flux per unit time (3.97)
M	Mach number (2.45)
\mathcal{M}	Mass fraction (2.30a)
Mo	Morton number (4.44)
MW	Molecular weight of a gas (1.24)
n	Wavenumber (6.92)
\mathbf{n}	Outward normal vector (2.1)
n_p	Number of particles per mixed-fluid volume (5.1)
N	Number of particles (5.1), realizations (5.26), nodes (6.106), or parcels (8.10)
\mathcal{N}	Number of collisions per unit volume of mixture (5.54)
NISI	No index summation intended
NTP	Normal temperature and pressure (Table A.1)
O	Order of a term
Oh	Ohnesorge number (4.54)
p	Continuous-phase pressure neglecting local flow around particle (1.15e)
P	Pressure around or inside of particle (1.12)
Pr	Prandtl number (2.29)
\mathcal{P}	Probability distribution function (4.1)
q	Arbitrary variable (1.7)
\mathbf{q}	Arbitrary vector (1.8)
\mathbf{Q}	Diffusive (nonconvective) flux (2.1)
\dot{Q}	Heat transfer rate (3.108)

r	Radial distance (1.11)
r	Dimensionless particle roughness (9.43b)
r_p	Volumetric particle radius (1.1b)
R	Acceleration parameter (3.88b)
\mathcal{R}	Gas constant (1.24b)
Re	Reynolds number (2.79)
s	Speed ratio (9.32)
s	Particle size ratio (11.45b)
S	Shape oscillation mode (4.70)
Sc	Schmidt number (2.35)
Sh	Sherwood number (3.103)
St	Stokes number (5.13)
t	Time (1.15)
t	Dimensionless turbulence intensity (9.43a)
T	Temperature (1.24)
\mathcal{T}	Torque (8.1c)
\mathbf{u}	Continuous-phase velocity neglecting local flow around the particle (1.15b)
\mathbf{U}	Continuous-phase velocity including local flow around the particle (1.12a)
\mathbf{v}	Velocity of the particle centroid (1.15a)
\mathbf{V}	Internal particle velocity field (1.12c)
\mathbf{w}	Relative velocity of the dispersed phase (1.15d)
\mathbf{W}	Faxen-corrected relative velocity (9.3f)
We	Weber number (4.41)
x	Streamwise direction for Cartesian coordinates (1.9)
\mathbf{X}	Position vector (1.9)
$\boldsymbol{\chi}$	Relative position vector (6.60)
x_i	General Cartesian direction (1.9)
y	Wall-normal direction for Cartesian coordinates (1.9)
Y	Drag-power parameter (4.18)
z	Spanwise direction for Cartesian coordinates (1.9)
Z	Transfer function (8.49)

Greek Letters and Other Symbols

α	Volume fraction (5.2)
β	Collision impact parameter (11.59)
δ	Kronecker delta (2.15b) or boundary-layer thickness (2.85b)
Δ	Discretization increment (space or time)
ε	Turbulent dissipation of continuous phase (6.33)
ϵ	Small perturbation ($\ll 1$)
ϕ	Azimuthal angle coordinate (1.11)
Φ	Velocity potential (2.57)
γ	Gas specific heat ratio (2.39)
Γ	Gamma function (4.27)
η	Kolmogorov length scale of the turbulence (6.86)

ϑ	Particle impact angle (11.59)
κ	Boltzmann constant (5.29b)
λ	Taylor length scale of the turbulence (6.95)
Λ	Integral length scale of the turbulence (6.71)
μ	Dynamic viscosity (1.6)
μ_p^*	Viscosity ratio (1.6)
Π	Multiphase coupling parameter (5.35)
ν	Kinematic viscosity (2.74)
θ	Polar angle for particle-centered coordinates (1.11)
Θ	Mass diffusivity (2.32)
Θ^*	Normalized turbulent particle diffusivity (7.17)
ρ	Density (1.4)
ρ_p^*	Density ratio (1.5)
σ	Surface tension (3.32)
\S	Section number
τ	Time scale (3.88a) or temporal shift (6.53)
Υ	Correlation coefficient (6.53)
∇	Volume (1.1)
ω	Continuous-phase fluid vorticity (2.51)
ω^*	Dimensionless continuous-phase vorticity (10.4a)
Ω	Angular rotation rate (Figure 10.1)
Ω^*	Dimensionless particle rotation rate (10.4b)
ψ	Stream function (2.50)
Ψ	Tangential velocity ratio (11.51)
ζ	Gaussian random number (8.64)
χ	Terminal velocity ratio (11.14)

Subscripts

all	All particles
avg	Average
@p	Continuous-fluid property extrapolated to particle center
b	Bin
body	Body force
buoy	Buoyancy effect
Br	Brownian motion
curv	Curvature
clean	Clean conditions where the interface is fully mobile
coll	Collisions
cont	Contaminated conditions
conv	Convective
crit	Critical condition where flow transitions
d	Volumetric diameter or dispersed phase
diss	Dissipation
dyn	Dynamic pressure

D	Drag or overall macroscopic domain scales
E	Particle aspect ratio
\mathcal{E}	Eulerian
eff	Effective
eq	Equilibrium
f	Continuous-phase fluid
fm	Free-molecular value
fr	Wall friction
ft	Frozen-turbulence limit
g	Gas
\mathbf{g}	Gravity
G	Subgrid filter
gap	Gap between particle surface and wall or another particle
H	History force effect
I	Interface between particle and surrounding fluid
inj	Injection
Kn	Knudsen
∇k	Due to gradients in the turbulent kinetic energy
l	Liquid or wavelength
L	Lift
\mathcal{L}	Lagrangian property
lam	Laminar
LN	Log-normal size distribution values
m	Mixed-fluid value
M	Mach number
m-m	Between two molecules
$m\mathcal{E}$	Moving Eulerian property
min	Minimum
n	Normal direction
nat	Natural frequency
o	Initial or reference state
osc	Related to oscillations
p	Particle phase (dispersed phase) or unhindered pressure
P	Parcel (group of particles) or local pressure
p/P	Particles per parcel (8.10b)
p,Δ	Particles that are within a computational cell (8.10a)
p-p	Particle–particle spacing (5.43)
plastic	Onset of plastic deformation
prod	Turbulent production
proj	Projected
pseudo	Pseudoturbulence
r	Radial direction
Re	Reynolds number effect
ref	Normal temperature and pressure conditions

rel	Local difference between particle and surrounding fluid
rms	Root of the mean of the squares (6.16)
RR	Rosin–Rammler size distribution values
S	Fluid-stress value
sd	Strong drift (7.19b)
sep	Separated wake conditions
shear	Linear velocity shear
sphere	For an equivalent spherical particle
stag	Stagnation conditions based on isentropic rest
sub	Subcritical conditions
super	Supercritical conditions
surf	Surface averaged quantity, typically of particle
t	Tangential
∇T	Due to gradients in temperature of continuous phase
term	Terminal characteristic of a particle in quiescent fluid
tot	Total conditions
trans	Transitional flow condition
turb	Turbulent
vapor	Vapor
viscous	Viscous
wall	Wall interaction effect
wd	Weak drift (7.19a)
yield	Yield stress value
α	Finite volume fraction effects
\forall	Added mass effect (3.76)
Δ	Cell resolution discretization
η	Kolmogorov scale of turbulence
Λ	Integral scale of turbulence
θ	Polar component
∞	Long-time or far-field property

Superscripts

in	Before interaction with wall
out	After interaction with wall
n	Time-step index
+	Normalized by wall shear-stress values
*	Dimensionless

Functions of an Arbitrary Property q

\hat{q}	Lagrangian path average (5.15)
\bar{q}	Eulerian time average (6.1)
q'	Instantaneous fluctuation from time average (6.4)
$\langle q \rangle$	Ensemble average for many particles (7.8a)
q''	Deviation from ensemble average (7.8b)

\bar{q}	Spatial averaged value for LES (6.44)
q	Spatial-filtered perturbation for LES (6.44)
∇'^2	Stokes specialized spherical operator (3.15)

Comparison Symbols

\equiv	Equal by definition
\approx	Approximately equal
\sim	Same order of magnitude
\lesssim	Approximately less than
\gtrsim	Approximately more than
\ll	Much smaller
\gg	Much larger
\perp	Normal component
\parallel	Parallel component
$O(q)$	On the order of q
$q \rightarrow 0$	q can be neglected