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## Theory of Vortex Sound

*Theory of Vortex Sound* is an introduction to the theory of sound generated by hydrodynamic flows. Starting with a review of elementary theoretical acoustics, the book proceeds to a unified treatment of low Mach number vortex-surface interaction noise in terms of the compact Green's function. Problems are provided at the end of each chapter, many of which can be used for extended student projects, and a whole chapter is devoted to worked examples.

It is designed for a one-semester introductory course at the advanced undergraduate or graduate levels. Great care is taken to explain underlying fluid mechanical and acoustic concepts, and to describe as fully as possible the steps in a complicated derivation.

M.S. Howe has been Professor in the Department of Aerospace and Mechanical Engineering at Boston University since 1992. He is a Fellow of the Institute of Acoustics (U.K.) and of the Acoustical Society of America.

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M. S. HOWE  
*Boston University*



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*To Shôn Ffowcs Williams*

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## *Preface*

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Vortex sound is the branch of fluid mechanics concerned with the conversion of hydrodynamic (*rotational*) kinetic energy into the longitudinal disturbances we call sound. The subject is itself a subsection of the theory of aerodynamic sound, which encompasses a much wider range of problems also involving, for example, combustion and ‘entropy’ sources of sound. The book is based on an introductory one-semester graduate level course given on several occasions at Boston University. Most students at this level possess an insufficient grasp of basic principles to appreciate the subtle coupling of the hydrodynamic and acoustic fields, and many are ill-equipped to deal with the novel analytical techniques that have been developed to investigate the coupling. Great care has therefore been taken to discuss underlying fluid mechanical and acoustic concepts, and to explain as fully as possible the steps in a complicated derivation.

A considerable number of practical problems occur at low Mach numbers (say, less than about 0.4). It seems reasonable, therefore, to confine an introductory discussion specifically to low Mach number flows. It is then possible to investigate a number of idealized hydrodynamic flows involving elementary distributions of vorticity adjacent to solid boundaries, and to analyze in detail the sound produced by these vortex–surface interactions. For a broad range of such problems, and a corresponding broad range of noise problems encountered in industrial applications, the effective acoustic sources turn out to be localized to one or more regions that are small compared to the acoustic wavelength. This permits the development of a unified theory of sound production by vortex–surface interactions in terms of the compact Green’s function, culminating in a routine procedure for estimating the sound, and providing, at the same time, an easy identification of those parts of a structure that are likely to be important sources of sound. Many examples of this type are discussed, and they are simple enough for the student to acquire an intuitive understanding of the method of

solution and the underlying physics. By these means the reader is encouraged to investigate both the hydrodynamics and the sound generated by a simple flow. Experience has shown that the successful completion of this kind of project, involving the implementation of a widely applicable yet standard procedure for the prediction of sound generation at low Mach numbers, motivates a student to understand the ostensibly difficult parts of the theory. One or more of the problems appended to some of the later chapters can form the basis of a project. The final chapter contains a set of worked examples that have been investigated by students at Boston University. I wish to thank my former students H. Abou-Hussein, A. DeBenedictis, N. Harrison, M. Kim, M. A. Rodrigues, and F. Zagadou for their considerable help in preparing that chapter.

The mathematical ability assumed of the reader is roughly equivalent to that taught in an advanced undergraduate course on Engineering Mathematics. In particular, the reader should be familiar with basic vector differential and integral calculus and with the repeated suffix summation convention of Cartesian tensors (but a detailed knowledge of tensor calculus is not required). An elementary understanding of the properties of the Dirac  $\delta$  function is desirable (Lighthill, 1958), including its interpretation as the formal limit of an  $\epsilon$ -sequence, such as

$$\delta(x) = \frac{\epsilon}{\pi(x^2 + \epsilon^2)}, \quad \epsilon \rightarrow +0.$$

Much use is made of the formula

$$\delta(f(x)) = \sum_n \frac{\delta(x - x_n)}{|f'(x_n)|},$$

where the summation is over real simple roots of  $f(x) = 0$ .

M. S. Howe