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0521812054 - Representations and Characters of Groups, Second Edition - Gordon James  
and Martin Liebeck

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## Representations and Characters of Groups

Now in its second edition, this text provides a modern introduction to the representation theory of finite groups. The authors have revised the popular first edition and added a considerable amount of new material. The theory is developed in terms of modules, since this is appropriate for more advanced work, but considerable emphasis is placed upon constructing characters. The character tables of many groups are given, including all groups of order less than 32, and all simple groups of order less than 1000.

Among the applications covered are Burnside's  $p^a q^b$  theorem, the use of character theory in studying subgroup structure and permutation groups, and a description of how to use representation theory to investigate molecular vibration.

Each chapter is accompanied by a variety of exercises, and full solutions to all the exercises are provided at the end of the book. This will be ideal as a text for a course in representation theory, and in view of the applications of the subject, will be of interest to mathematicians, chemists and physicists alike.

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# REPRESENTATIONS AND CHARACTERS OF GROUPS

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*Department of Mathematics,  
Imperial College, London*

Second Edition



**CAMBRIDGE**  
UNIVERSITY PRESS

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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS  
The Edinburgh Building, Cambridge CB2 2RU, UK  
40 West 20th Street, New York, NY 10011-4211, USA  
10 Stamford Road, Oakleigh, VIC 3166, Australia  
Ruiz de Alarcón 13, 28014 Madrid, Spain  
Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

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First published 1993

Reprinted 1995

Second edition 2001

Printed in the United Kingdom at the University Press, Cambridge

*Typeface* Times 10/13pt. *System* 3B2 [KT]

*A catalogue record for this book is available from the British Library*

ISBN 0 521 00392 X paperback

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## Preface

We have attempted in this book to provide a leisurely introduction to the representation theory of groups. But why should this subject interest you?

Representation theory is concerned with the ways of writing a group as a group of matrices. Not only is the theory beautiful in its own right, but it also provides one of the keys to a proper understanding of finite groups. For example, it is often vital to have a concrete description of a particular group; this is achieved by finding a representation of the group as a group of matrices. Moreover, by studying the different representations of the group, it is possible to prove results which lie outside the framework of representation theory. One simple example: all groups of order  $p^2$  (where  $p$  is a prime number) are abelian; this can be shown quickly using only group theory, but it is also a consequence of basic results about representations. More generally, all groups of order  $p^a q^b$  ( $p$  and  $q$  primes) are soluble; this again is a statement purely about groups, but the best proof, due to Burnside, is an outstanding example of the use of representation theory. In fact, the range of applications of the theory extends far beyond the boundaries of pure mathematics, and includes theoretical physics and chemistry – we describe one such application in the last chapter.

The book is suitable for students who have taken first undergraduate courses involving group theory and linear algebra. We have included two preliminary chapters which cover the necessary background material. The basic theory of representations is developed in Chapters 3–23, and our methods concentrate upon the use of modules; although this accords with the more modern style of algebra, in several instances our proofs differ from those found in other textbooks. The main results are elegant and surprising, but at first sight they sometimes have an air of mystery

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*Representations and characters of groups*

about them; we have chosen the approach which we believe to be the most transparent.

We also emphasize the practical aspects of the subject, and the text is illustrated with a wealth of examples. A feature of the book is the wide variety of groups which we investigate in detail. By the end of Chapter 28, we have presented the character tables of all groups of order less than 32, of all  $p$ -groups of order at most  $p^4$ , and of all the simple groups of order less than 1000.

Every chapter is accompanied by a set of Exercises, and the solutions to all of these are provided at the end of the book.

We would like to thank Dr Hans Liebeck for his careful reading of our manuscript and the many helpful suggestions which he made.

*Preface to Second Edition*

In this second edition, we have included two new chapters; one (Chapter 28) deals with the character tables of an infinite series of groups, and the other (Chapter 29) covers aspects of the representation theory of permutation groups. We have also added a considerable amount of new material to Chapters 20, 23 and 30, and made minor amendments elsewhere.