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978-0-521-81129-3 - Stochastic Integration with Jumps
Klaus Bichteler
Frontmatter
More information

Stochastic Integration with Jumps

Stochastic processes with jumps and random measures are gaining importance as drivers in applications like financial mathematics and signal processing. This book develops the stochastic integration theory for both integrators (semimartingales) and random measures from a common point of view.

Highlights feature the DCT and Egoroff's theorem, as well as comprehensive analogs to results from ordinary integration theory, for instance, previsible envelopes and an algorithm computing the stochastic integrals of càglàd integrands pathwise.

An integrator under P is continuous as a map into L^q for any finite q, provided that P is replaced with a suitable probability, and there is control of the transition; this extends to random measures when $q = 2$. This has the consequence that every integrator is controlled by some previsible process in much the same way a Wiener process is controlled by time t. The previsible controller furnishes Picard norms that reduce SDEs to simple (global) fixed-point problems with an easy stability theory and numerical pathwise approximation schemes.

Full proofs are given for all results, and motivation is stressed throughout. A large appendix contains most of the analysis that readers will need as a prerequisite. A comprehensive reference list and an index of notation are also provided. Extra material is available from the book's Web site at http://www.ma.utexas.edu/users/cup.

This will be an invaluable reference for graduate students and researchers in mathematics, physics, electrical engineering, and finance who need to use stochastic differential equations.

Klaus Bichteler is a Professor of Mathematics at the University of Texas at Austin. He received his Ph.D. in physics from Hamburg University in 1965. He has written extensively on general relativity, representation theory, integration, probability, and Malliavin calculus.

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

Volume 89
Stochastic Integration with Jumps

27 N. H. Bingham, C. M. Goldie and J. L. Teugels *Regular Variation*
29 N. White (ed.) *Combinatorial Geometries*
30 M. Pohst and H. Zassenhaus *Algorithmic Algebraic Number Theory*
31 J. Aczel and J. Dhombres *Functional Equations in Several Variables*
32 M. Kuczma, B. Choczewski and R. Ger *Iterative Functional Equations*
33 R. V. Ambartzumian *Factorization Calculus and Geometric Probability*
34 G. Gripenberg, S.-O. Londen and O. Staffans *Volterra Integral and Functional Equations*
35 G. Gasper and M. Rahman *Basic Hypergeometric Series*
36 E. Torgersen *Comparison of Statistical Experiments*
38 N. Korneichuk *Exact Constants in Approximation Theory*
39 R. Brualdi and H. Ryser *Combinatorial Matric Theory*
40 N. White (ed.) *Matroid Applications*
41 S. Sakai *Operator Algebras in Dynamical Systems*
42 W. Hodges *Basic Model Theory*
43 H. Stahl and V. Totik *General Orthogonal Polynomials*
45 G. Da Prato and J. Zabczyk *Stochastic Equations in Infinite Dimensions*
46 A. Björner et al. *Oriented Matroids*
47 G. Edgar and L. Sucheston *Stopping Times and Directed Processes*
48 C. Sims *Computation with Finitely Presented Groups*
49 T. Palmer *Banach Algebras and the General Theory of *-Algebras I*
50 F. Borceux *Handbook of Categorical Algebra I*
51 F. Borceux *Handbook of Categorical Algebra II*
52 F. Borceux *Handbook of Categorical Algebra III*
53 V. F. Kolchin *Random Graphics*
54 A. Katok and B. Hasselblatt *Introduction to the Modern Theory of Dynamical Systems*
55 V. N. Sachkov *Combinatorial Methods in Discrete Mathematics*
56 V. N. Sachkov *Probabilistic Methods in Discrete Mathematics*
57 M. Cohn *Skew Fields*
58 R. Gardner *Geometric Topography*
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60 J. Krajicek *Bounded Arithmetic Propositional Logic and Complexity Theory*
61 H. Groemer *Geometric Applications of Fourier Series and Spherical Harmonics*
62 H. O. Fattorini *Infinite Dimensional Optimization and Control Theory*
63 A. C. Thompson *Minkowski Geometry*
64 R. B. Bapat and T. E. S. Raghavan *Nonnegative Matrices with Applications*
65 K. Engel *Sperner Theory*
66 D. Cvetkovic, P. Rowlinson and S. Simic *Eigenspaces of Graphs*
67 F. Bergeron, G. Labelle and P. Leroux *Combinatorial Species and Tree-Like Structures*
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71 G. E. Andrews, R. Askey and R. Roy *Special Functions*
72 R. Ticciati *Quantum Field Theory for Mathematicians*
73 M. Stern *Semimodular Lattices*
74 I. Lasiecka and R. Triggiani *Control Theory for Partial Differential Equations I*
75 I. Lasiecka and R. Triggiani *Control Theory for Partial Differential Equations II*
76 A. A. Ivanov *Geometry of Sporadic Groups 1*
77 A. Schinzel *Polynomials with Special Regard to Reducibility*
78 H. Lenz, T. Beth and D. Jungnickel *Design Theory II, 2nd Edition*
79 T. Palmer *Banach Algebras and the General Theory of *-Algebras II*
80 O. Stormark *Lie's Structural Approach to PDE Systems*
81 C. F. Dunkl and Y. Xu *Orthogonal Polynomials of Several Variables*
82 J. P. Mayberry *The Foundations of Mathematics in the Theory of Sets*

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In memoriam
Charlotte Rosa Luise

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Contents

Preface

This book originated with several courses given at the University of Texas. The audience consisted of graduate students of mathematics, physics, electrical engineering, and finance. Most had met some stochastic analysis during work in their field; the course was meant to provide the mathematical underpinning. To satisfy the economists, driving processes other than Wiener process had to be treated; to give the mathematicians a chance to connect with the literature and discrete-time martingales, I chose to include driving terms with jumps. This plus a predilection for generality for simplicity's sake led directly to the most general stochastic Lebesgue–Stieltjes integral.

The spirit of the exposition is as follows: just as having finite variation and being right-continuous identifies the useful Lebesgue–Stieltjes distribution functions among all functions on the line, are there criteria for processes to be useful as "random distribution functions." They turn out to be straightforward generalizations of those on the line. A process that meets these criteria is called an *integrator*, and its integration theory is just as easy as that of a deterministic distribution function on the line – provided Daniell's method is used. (This proviso has to do with the lack of convexity in some of the target spaces of the stochastic integral.)

For the purpose of error estimates in approximations both to the stochastic integral and to solutions of stochastic differential equations we define various *numerical sizes of an integrator* Z and analyze rather carefully how they propagate through many operations done on and with Z, for instance, solving a stochastic differential equation driven by Z. These size-measurements arise as generalizations to integrators of the famed Burkholder–Davis–Gundy inequalities for martingales. The present exposition differs in the ubiquitous use of numerical estimates from the many fine books on the market, where convergence arguments are usually done in probability or every once in a while in Hilbert space L^2. For reasons that unfold with the story we employ the L^p-norms in the whole range $0 \le p < \infty$. An effort is made to furnish reasonable estimates for the universal constants that occur in this context.

Such attention to estimates, unusual as it may be for a book on this subject, pays handsomely with some new results that may be edifying even to the expert. For instance, it turns out that every integrator Z can be controlled by an increasing previsible process much like a Wiener process is controlled by time t; and if not with respect to the given probability, then at least with respect to an equivalent one that lets one view the given integrator as a map into Hilbert space, where computation is comparatively facile. This *previsible controller* obviates prelocal arguments [91] and can be used to construct Picard norms for the solution of stochastic differential equations driven by Z that allow growth estimates, easy treatment of stability theory, and even *pathwise algorithms* for the solution. These schemes extend without ado to *random measures*, including the previsible control and its application to stochastic differential equations driven by them.

All this would seem to lead necessarily to an enormous number of technicalities. A strenuous effort is made to keep them to a minimum, by these devices: everything not directly needed in stochastic integration theory and its application to the solution of stochastic differential equations is either omitted or relegated to the Supplements or to the Appendices. A short survey of the beautiful "General Theory of Processes" developed by the French school can be found there.

A warning concerning the usual conditions is appropriate at this point. They have been replaced throughout with what I call the *natural conditions*. This will no doubt arouse the ire of experts who think one should not "tamper with a mature field." However, many fine books contain erroneous statements of the important Girsanov theorem – in fact, it is hard to find a correct statement in unbounded time – and this is traceable directly to the employ of the usual conditions (see example 3.9.14 on page 164 and 3.9.20). In mathematics, correctness trumps conformity. The natural conditions confer the same benefits as do the usual ones: path regularity (section 2.3), section theorems (page 437 ff.), and an ample supply of stopping times (*ibidem*), without setting a trap in Girsanov's theorem.

The students were expected to know the basics of point set topology up to Tychonoff's theorem, general integration theory, and enough functional analysis to recognize the Hahn–Banach theorem. If a fact fancier than that is needed, it is provided in appendix A, or at least a reference is given.

The exercises are sprinkled throughout the text and form an integral part. They have the following appearance:

Exercise 4.3.2 This is an exercise. It is set in a smaller font. It requires no novel argument to solve it, only arguments and results that have appeared earlier. Answers to some of the exercises can be found in appendix B. Answers to most of them can be found in appendix C, which is available on the web *via* `http://www.ma.utexas.edu/users/cup/Answers`.

I made an effort to index every technical term that appears (page 489), and to make an index of notation that gives a short explanation of every symbol and lists the page where it is defined in full (page 483). Both indexes appear in expanded form at `http://www.ma.utexas.edu/users/cup/Indexes`.

http://www.ma.utexas.edu/users/cup/Errata contains the errata. I plead with the gentle reader to send me the errors he/she found via email to kbi@math.utexas.edu, so that I may include them, with proper credit of course, in these errata.

At this point I recommend reading the conventions on page 363.