CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

Contents

Preface *page* xvii

1 Introduction **1**
1.1 Conservation Laws 3
1.2 Finite Volume Methods 5
1.3 Multidimensional Problems 6
1.4 Linear Waves and Discontinuous Media 7
1.5 CLAWPACK Software 8
1.6 References 9
1.7 Notation 10

Part I Linear Equations

2 Conservation Laws and Differential Equations **15**
2.1 The Advection Equation 17
2.2 Diffusion and the Advection–Diffusion Equation 20
2.3 The Heat Equation 21
2.4 Capacity Functions 22
2.5 Source Terms 22
2.6 Nonlinear Equations in Fluid Dynamics 23
2.7 Linear Acoustics 26
2.8 Sound Waves 29
2.9 Hyperbolicity of Linear Systems 31
2.10 Variable-Coefficient Hyperbolic Systems 33
2.11 Hyperbolicity of Quasilinear and Nonlinear Systems 34
2.12 Solid Mechanics and Elastic Waves 35
2.13 Lagrangian Gas Dynamics and the p-System 41
2.14 Electromagnetic Waves 43
Exercises 46

3 Characteristics and Riemann Problems for Linear Hyperbolic Equations **47**
3.1 Solution to the Cauchy Problem 47

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

3.2 Superposition of Waves and Characteristic Variables 48
3.3 Left Eigenvectors 49
3.4 Simple Waves 49
3.5 Acoustics 49
3.6 Domain of Dependence and Range of Influence 50
3.7 Discontinuous Solutions 52
3.8 The Riemann Problem for a Linear System 52
3.9 The Phase Plane for Systems of Two Equations 55
3.10 Coupled Acoustics and Advection 57
3.11 Initial–Boundary-Value Problems 59
Exercises 62

4 Finite Volume Methods 64
4.1 General Formulation for Conservation Laws 64
4.2 A Numerical Flux for the Diffusion Equation 66
4.3 Necessary Components for Convergence 67
4.4 The CFL Condition 68
4.5 An Unstable Flux 71
4.6 The Lax–Friedrichs Method 71
4.7 The Richtmyer Two-Step Lax–Wendroff Method 72
4.8 Upwind Methods 72
4.9 The Upwind Method for Advection 73
4.10 Godunov's Method for Linear Systems 76
4.11 The Numerical Flux Function for Godunov's Method 78
4.12 The Wave-Propagation Form of Godunov's Method 78
4.13 Flux-Difference vs. Flux-Vector Splitting 83
4.14 Roe's Method 84
Exercises 85

5 Introduction to the CLAWPACK Software 87
5.1 Basic Framework 87
5.2 Obtaining CLAWPACK 89
5.3 Getting Started 89
5.4 Using CLAWPACK – a Guide through `example1` 91
5.5 Other User-Supplied Routines and Files 98
5.6 Auxiliary Arrays and `setaux.f` 98
5.7 An Acoustics Example 99
Exercises 99

6 High-Resolution Methods 100
6.1 The Lax–Wendroff Method 100
6.2 The Beam–Warming Method 102
6.3 Preview of Limiters 103
6.4 The REA Algorithm with Piecewise Linear Reconstruction 106

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

6.5	Choice of Slopes	107
6.6	Oscillations	108
6.7	Total Variation	109
6.8	TVD Methods Based on the REA Algorithm	110
6.9	Slope-Limiter Methods	111
6.10	Flux Formulation with Piecewise Linear Reconstruction	112
6.11	Flux Limiters	114
6.12	TVD Limiters	115
6.13	High-Resolution Methods for Systems	118
6.14	Implementation	120
6.15	Extension to Nonlinear Systems	121
6.16	Capacity-Form Differencing	122
6.17	Nonuniform Grids	123
	Exercises	127
7	**Boundary Conditions and Ghost Cells**	**129**
7.1	Periodic Boundary Conditions	130
7.2	Advection	130
7.3	Acoustics	133
	Exercises	138
8	**Convergence, Accuracy, and Stability**	**139**
8.1	Convergence	139
8.2	One-Step and Local Truncation Errors	141
8.3	Stability Theory	143
8.4	Accuracy at Extrema	149
8.5	Order of Accuracy Isn't Everything	150
8.6	Modified Equations	151
8.7	Accuracy Near Discontinuities	155
	Exercises	156
9	**Variable-Coefficient Linear Equations**	**158**
9.1	Advection in a Pipe	159
9.2	Finite Volume Methods	161
9.3	The Color Equation	162
9.4	The Conservative Advection Equation	164
9.5	Edge Velocities	169
9.6	Variable-Coefficient Acoustics Equations	171
9.7	Constant-Impedance Media	172
9.8	Variable Impedance	173
9.9	Solving the Riemann Problem for Acoustics	177
9.10	Transmission and Reflection Coefficients	178
9.11	Godunov's Method	179
9.12	High-Resolution Methods	181

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

9.13 Wave Limiters 181
9.14 Homogenization of Rapidly Varying Coefficients 183
Exercises 187

10 Other Approaches to High Resolution 188
10.1 Centered-in-Time Fluxes 188
10.2 Higher-Order High-Resolution Methods 190
10.3 Limitations of the Lax–Wendroff (Taylor Series) Approach 191
10.4 Semidiscrete Methods plus Time Stepping 191
10.5 Staggered Grids and Central Schemes 198
Exercises 200

Part II Nonlinear Equations

11 Nonlinear Scalar Conservation Laws 203
11.1 Traffic Flow 203
11.2 Quasilinear Form and Characteristics 206
11.3 Burgers' Equation 208
11.4 Rarefaction Waves 209
11.5 Compression Waves 210
11.6 Vanishing Viscosity 210
11.7 Equal-Area Rule 211
11.8 Shock Speed 212
11.9 The Rankine–Hugoniot Conditions for Systems 213
11.10 Similarity Solutions and Centered Rarefactions 214
11.11 Weak Solutions 215
11.12 Manipulating Conservation Laws 216
11.13 Nonuniqueness, Admissibility, and Entropy Conditions 216
11.14 Entropy Functions 219
11.15 Long-Time Behavior and N-Wave Decay 222
Exercises 224

12 Finite Volume Methods for Nonlinear Scalar Conservation Laws 227
12.1 Godunov's Method 227
12.2 Fluctuations, Waves, and Speeds 229
12.3 Transonic Rarefactions and an Entropy Fix 230
12.4 Numerical Viscosity 232
12.5 The Lax–Friedrichs and Local Lax–Friedrichs Methods 232
12.6 The Engquist–Osher Method 234
12.7 E-schemes 235
12.8 High-Resolution TVD Methods 235
12.9 The Importance of Conservation Form 237
12.10 The Lax–Wendroff Theorem 239

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

12.11 The Entropy Condition 243
12.12 Nonlinear Stability 244
Exercises 252

13 Nonlinear Systems of Conservation Laws 253
13.1 The Shallow Water Equations 254
13.2 Dam-Break and Riemann Problems 259
13.3 Characteristic Structure 260
13.4 A Two-Shock Riemann Solution 262
13.5 Weak Waves and the Linearized Problem 263
13.6 Strategy for Solving the Riemann Problem 263
13.7 Shock Waves and Hugoniot Loci 264
13.8 Simple Waves and Rarefactions 269
13.9 Solving the Dam-Break Problem 279
13.10 The General Riemann Solver for Shallow Water Equations 281
13.11 Shock Collision Problems 282
13.12 Linear Degeneracy and Contact Discontinuities 283
Exercises 287

14 Gas Dynamics and the Euler Equations 291
14.1 Pressure 291
14.2 Energy 292
14.3 The Euler Equations 293
14.4 Polytropic Ideal Gas 293
14.5 Entropy 295
14.6 Isothermal Flow 298
14.7 The Euler Equations in Primitive Variables 298
14.8 The Riemann Problem for the Euler Equations 300
14.9 Contact Discontinuities 301
14.10 Riemann Invariants 302
14.11 Solution to the Riemann Problem 302
14.12 The Structure of Rarefaction Waves 305
14.13 Shock Tubes and Riemann Problems 306
14.14 Multifluid Problems 308
14.15 Other Equations of State and Incompressible Flow 309

15 Finite Volume Methods for Nonlinear Systems 311
15.1 Godunov's Method 311
15.2 Convergence of Godunov's Method 313
15.3 Approximate Riemann Solvers 314
15.4 High-Resolution Methods for Nonlinear Systems 329
15.5 An Alternative Wave-Propagation Implementation of Approximate Riemann Solvers 333
15.6 Second-Order Accuracy 335

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

15.7 Flux-Vector Splitting 338
15.8 Total Variation for Systems of Equations 340
Exercises 348

16 Some Nonclassical Hyperbolic Problems 350
16.1 Nonconvex Flux Functions 350
16.2 Nonstrictly Hyperbolic Problems 358
16.3 Loss of Hyperbolicity 362
16.4 Spatially Varying Flux Functions 368
16.5 Nonconservative Nonlinear Hyperbolic Equations 371
16.6 Nonconservative Transport Equations 372
Exercises 374

17 Source Terms and Balance Laws 375
17.1 Fractional-Step Methods 377
17.2 An Advection–Reaction Equation 378
17.3 General Formulation of Fractional-Step Methods for Linear Problems 384
17.4 Strang Splitting 387
17.5 Accuracy of Godunov and Strang Splittings 388
17.6 Choice of ODE Solver 389
17.7 Implicit Methods, Viscous Terms, and Higher-Order Derivatives 390
17.8 Steady-State Solutions 391
17.9 Boundary Conditions for Fractional-Step Methods 393
17.10 Stiff and Singular Source Terms 396
17.11 Linear Traffic Flow with On-Ramps or Exits 396
17.12 Rankine–Hugoniot Jump Conditions at a Singular Source 397
17.13 Nonlinear Traffic Flow with On-Ramps or Exits 398
17.14 Accurate Solution of Quasisteady Problems 399
17.15 Burgers Equation with a Stiff Source Term 401
17.16 Numerical Difficulties with Stiff Source Terms 404
17.17 Relaxation Systems 410
17.18 Relaxation Schemes 415
Exercises 416

Part III Multidimensional Problems

18 Multidimensional Hyperbolic Problems 421
18.1 Derivation of Conservation Laws 421
18.2 Advection 423
18.3 Compressible Flow 424
18.4 Acoustics 425
18.5 Hyperbolicity 425
18.6 Three-Dimensional Systems 428
18.7 Shallow Water Equations 429

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

18.8	Euler Equations	431
18.9	Symmetry and Reduction of Dimension	433
	Exercises	434
19	**Multidimensional Numerical Methods**	**436**
19.1	Finite Difference Methods	436
19.2	Finite Volume Methods and Approaches to Discretization	438
19.3	Fully Discrete Flux-Differencing Methods	439
19.4	Semidiscrete Methods with Runge–Kutta Time Stepping	443
19.5	Dimensional Splitting	444
	Exercise	446
20	**Multidimensional Scalar Equations**	**447**
20.1	The Donor-Cell Upwind Method for Advection	447
20.2	The Corner-Transport Upwind Method for Advection	449
20.3	Wave-Propagation Implementation of the CTU Method	450
20.4	von Neumann Stability Analysis	452
20.5	The CTU Method for Variable-Coefficient Advection	453
20.6	High-Resolution Correction Terms	456
20.7	Relation to the Lax–Wendroff Method	456
20.8	Divergence-Free Velocity Fields	457
20.9	Nonlinear Scalar Conservation Laws	460
20.10	Convergence	464
	Exercises	467
21	**Multidimensional Systems**	**469**
21.1	Constant-Coefficient Linear Systems	469
21.2	The Wave-Propagation Approach to Accumulating Fluxes	471
21.3	CLAWPACK Implementation	473
21.4	Acoustics	474
21.5	Acoustics in Heterogeneous Media	476
21.6	Transverse Riemann Solvers for Nonlinear Systems	480
21.7	Shallow Water Equations	480
21.8	Boundary Conditions	485
22	**Elastic Waves**	**491**
22.1	Derivation of the Elasticity Equations	492
22.2	The Plane-Strain Equations of Two-Dimensional Elasticity	499
22.3	One-Dimensional Slices	502
22.4	Boundary Conditions	502
22.5	The Plane-Stress Equations and Two-Dimensional Plates	504
22.6	A One-Dimensional Rod	509
22.7	Two-Dimensional Elasticity in Heterogeneous Media	509

© Cambridge University Press www.cambridge.org

CAMBRIDGE

Cambridge University Press
0521810876 - Finite Volume Methods for Hyperbolic Problems - Randall J. LeVeque
Table of Contents
More information

23 Finite Volume Methods on Quadrilateral Grids **514**
23.1 Cell Averages and Interface Fluxes 515
23.2 Logically Rectangular Grids 517
23.3 Godunov's Method 518
23.4 Fluctuation Form 519
23.5 Advection Equations 520
23.6 Acoustics 525
23.7 Shallow Water and Euler Equations 530
23.8 Using CLAWPACK on Quadrilateral Grids 531
23.9 Boundary Conditions 534

Bibliography 535

Index 553

© Cambridge University Press www.cambridge.org