1
Overview and overture

Einstein's theory of the classical relativistic dynamics of gravity is remarkable, both in its simple elegance and in its profound statement about the nature of spacetime. Before we rush into the diverse matters which concern and motivate the search which leads to string theory and beyond, such as the nature of the quantum theory, the unification with other forces, etc., let us remind ourselves of some of the salient features of the classical theory. This will usefully foreshadow many of the concepts which we will encounter later.

1.1 The classical dynamics of geometry

Spacetime is of course a landscape of 'events', the points which make it up, and as such it is a classical (but of course relativistic) concept. Intuition from quantum mechanics points to a modification of this picture, and there are many concrete mechanisms in string theory which support this expectation and show that spacetime is at best a derived object or effective description. We shall see some of these mechanisms in the sequel. However, since string theory (as currently understood), seems to be devoid of a complete definition that does not require us to refer to spacetime, the language and concepts we will employ will have much in common with those used by professional practitioners of General Relativity, and of classical and quantum Field Theory. In fact, it will become clear to the newcomer that success in the physics of string theory is greatly aided by having technical facility in both of those fields. It is instructive to tour a little of the foundations of our modern approach to classical gravity and observe how the Relativist's and the Field Theorist's perspective are muddled together. String theory makes good and productive use of this sort of conflation.

It is useful to equip a description of spacetime with a set of coordinates x^μ, $\mu = 0, 1, \ldots, D-1$, where $x^0 \equiv t$ (the time) and we shall remain open-minded and work in D dimensions for much of the discussion. The metric, with components $g_{\mu\nu}(x)$, is a function of the coordinates which allows for a local measure of the distance between points separated by an interval dx^μ:

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu.$$

The metric is a tensor field since under an arbitrary change of variables $x^\mu \to x'^\mu(x)$ it transforms as

$$g_{\mu\nu} \longrightarrow g'_{\mu\nu} = g_{\alpha\beta}\frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}. \qquad (1.1)$$

Of course, 'distance' here means the more generalised Special Relativistic interval characterising how two events are separated, and it is negative, zero or positive, giving us timelike, null or spacelike separations, according to whether if it possible to connect the events by causal subluminal motion (appropriate to a massive particle), or by moving at the speed of light (massless particles), or not. This of course defines the signature of our metric as being 'mostly plus': $\{- + + + \cdots\}$ henceforth.

As a particle moves it sweeps out a path or 'world-line' $x^\mu(\tau)$ in spacetime (see figure 1.1), which is parametrised by τ. The wonderful thing is that what we would have said in pre-Einstein times was 'a particle moving under the influence of the gravitational force' is simply replaced by the statement 'a particle following a geodesic', a path which is determined by the metric in terms of the second order geodesic equation:

$$\frac{d^2x^\lambda}{d\tau^2} = -\Gamma^\lambda_{\mu\nu}(g)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\ , \qquad (1.2)$$

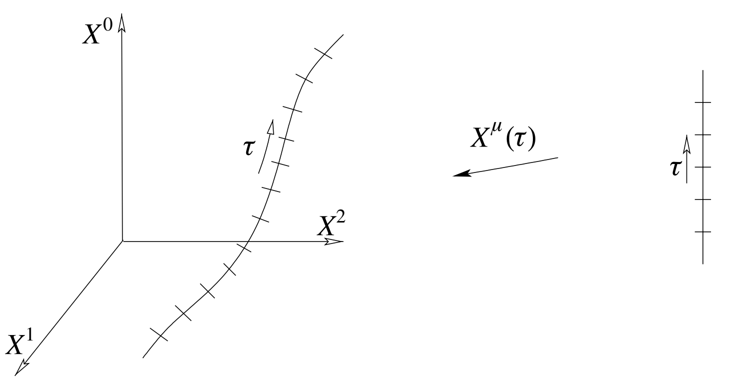

Fig. 1.1. A particle's world-line. The function $x^\mu(\tau)$ embeds the world-line, parametrised by τ, into spacetime, coordinatised by x^μ.

where the affine connection $\Gamma(g)$ is made out of first derivatives of the metric:

$$\Gamma^\lambda_{\mu\nu}(g) = \frac{1}{2}g^{\lambda\kappa}\left(\partial_\mu g_{\kappa\nu} + \partial_\nu g_{\kappa\mu} - \partial_\kappa g_{\mu\nu}\right).$$

Here and everywhere else, when working with curved spacetime we lower and raise indices with the metric and its inverse, (which has components $g^{\mu\nu}$ such that $g_{\mu\lambda}g^{\mu\alpha} = \delta^\alpha_\lambda$). Also note that $\partial_\mu \equiv \partial/\partial x^\mu$.

Switching language again we see that since the term on the left hand side of the equation (1.2) is what we think of as the 'acceleration', our Newtonian intuition determines the right hand side to be the 'applied force', attributed to gravity. In such language, $g_{\mu\nu}(x)$ is interpreted as a potential for the gravitational field.

In the purely geometrical language, there are no forces. There is only geometry, and the particle simply moves along geodesics. The above statement in equation (1.2) about how a particle moves in response to the metric is derivable from a simple action principle, which says that the motion minimises (more properly, extremises) the total path length that its motion sweeps out:

$$S = -m\int(-g_{\mu\nu}(x)dx^\mu dx^\nu)^{1/2} = -m\int_{\tau_\mathrm{i}}^{\tau_\mathrm{f}}(-g_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu)^{1/2}d\tau\ , \qquad (1.3)$$

where a dot denotes a derivative with respect to τ. (The reader might consider checking this by application of the Euler–Lagrange equations or by direct variation.)

The only question (which is of course one of the biggest) remaining is the nature of what determines the metric itself. This turns out to be governed by the distribution of stress-energy-momentum, and we must write field equations which determine how the one sources the other, just as we would in any field theory like Maxwell's electromagnetism (see insert 1.1).

The stress-energy-momentum contained in the matter is captured in the elegant package that is the tensor $T^{\mu\nu}(x)$, a second rank, symmetric, divergence-free tensor which for an observer with four-velocity $\mathbf{u}$, encodes the energy density as $T_{\mu\nu}u^\mu u^\nu$, the momentum density as $-T_{\mu\nu}u^\mu x^\nu$, and shear pressures (stresses) as $T_{\mu\nu}x^\mu y^\nu$, where the unit vectors $\mathbf{x}$ and $\mathbf{y}$ are orthogonal to $\mathbf{u}$.

Einstein's field equations are:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_\mathrm{N}T_{\mu\nu}\ , \qquad (1.6)$$

where G_N is Newton's constant. As one would expect, the quantity on the left hand side is made up of the metric and its first and second derivatives,

Insert 1.1. A reminder of Maxwell's field equations

'Maxwell's equations' are second order partial differential equations for the electromagnetic potentials $\vec{A}\,(\vec{x}, t), \phi(\vec{x}, t)$ from which the magnetic $(\vec{B}\,(\vec{x}, t))$ and electric $(\vec{E}\,(\vec{x}, t))$ fields can be derived:

$$\begin{aligned}\vec{E}\,(\vec{x}, t) &= -\vec{\nabla}\phi(\vec{x}\cdot t) - \frac{\partial\vec{A}\,(\vec{x}, t)}{\partial t}\\ \vec{B}\,(\vec{x}, t) &= \vec{\nabla}\times\vec{A}\,(\vec{x}, t).\end{aligned} \tag{1.4}$$

In terms of the fields, Maxwell's equations are:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E} &= 4\pi\rho\\ \vec{\nabla}\cdot\vec{B} &= 0\\ \vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t} &= 0\\ \vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t} &= 4\pi\,\vec{J}\,.\end{aligned} \tag{1.5}$$

Here, the functions $\vec{J}(\vec{x}, t)$ and $\rho(\vec{x}, t)$, the *current density* and the *charge density* are the *'sources'* in the field equations.

We have written the equations with the sources on the right hand side and the expression for the derivatives of the resulting fields (to which the sources give rise) on the left hand side. We will write these much more covariantly in insert 1.3.

where the Ricci scalar and tensor,

$$R \equiv g^{\mu\nu}R_{\mu\nu}, \qquad R_{\mu\nu} \equiv g^{\kappa\rho}g_{\lambda\rho}R^{\lambda}_{\mu\kappa\nu}, \tag{1.7}$$

are the only two contentful contractions of the Riemann tensor:

$$R^{\lambda}_{\mu\kappa\nu} \equiv \partial_\mu\Gamma^{\lambda}_{\kappa\nu} - \partial_\nu\Gamma^{\lambda}_{\kappa\mu} + \Gamma^{\rho}_{\kappa\mu}\Gamma^{\lambda}_{\rho\nu} - \Gamma^{\rho}_{\kappa\nu}\Gamma^{\lambda}_{\rho\mu}. \tag{1.8}$$

Except for the metric itself, the quantity on the left hand side of equation (1.6) is the unique rank two, divergenceless and symmetric tensor made from the metric (and its first and second derivatives), and hence can be allowed to be equated to the stress tensor.

When the stress tensor is zero, i.e. when there is no matter to act as a source, the vanishing of the left hand side is equivalent to the vanishing

$R_{\mu\nu} = 0$, and solutions of this equation are said to be 'Ricci-flat'. This includes highly non-trivial spacetimes such as Schwarzschild black holes, which follows from the non-linearity of the left hand side, representing the fact that the stress-energy in the gravitational field itself can act as its own source ('gravity gravitates').

The physical foundation behind the geometric approach is of course the Principle of Equivalence, which begins by observing that gravity is indistinguishable from acceleration, and tells one how to find a locally inertial frame: one must simply 'fall' under the influence of gravity (i.e. just follow a geodesic) and one does not feel one's own weight, and so one is in an inertial frame where the Laws of Special Relativity hold. See insert 1.2 for a reminder of this in equations. The sourceless field equations then follow from the recasting of the relative motion observed between frames on neighbouring geodesics in terms of an apparent 'tidal' force.

The full statement of the field equations to include sources is also guided by covariance, which means that it is a physical equation between tensors of the same type, and with the same divergenceless property (which is a physical statement of continuity). The equations are therefore true in all coordinate systems obtained by an arbitrary change of variables $x^\mu \to x'^\mu(x)$, since they transform as tensors in a way generalising the transformation of the metric in equation (1.1).

Note that the statement of divergencelessness is a covariant one too, i.e. $\nabla_\mu T^{\mu\nu} = 0$ uses the covariant derivative*, which is designed to yield a tensor after acting on one, say V:

$$\nabla_\kappa V^{\mu\cdots}_{\nu\cdots} \equiv \partial_\kappa V^{\mu\cdots}_{\nu\cdots} + \Gamma^\mu_{\lambda\kappa} V^{\lambda\cdots}_{\nu\cdots} + \cdots - \Gamma^\lambda_{\kappa\nu} V^{\mu\cdots}_{\lambda\cdots} - \cdots. \tag{1.9}$$

Finally, note that the field equations themselves may be derived from an action principle, the extremising of the Einstein–Hilbert action coupled to matter:

$$\begin{aligned} S &= S_{\rm M} + S_{\rm EH} \\ S_{\rm EH} &= \frac{1}{16\pi G_{\rm N}} \int d^D x \sqrt{-g}\ R \\ T^{\mu\nu} &\equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{\rm M}}{\delta g_{\mu\nu}}, \end{aligned} \tag{1.10}$$

where g is the determinant of the metric.

* In fact, this (not entirely unambiguous) procedure of replacing the ordinary derivative by the covariant derivative, together with the replacement of the Minkowski metric $\eta_{\mu\nu}$ by the curved spacetime metric $g_{\mu\nu}(x)$ is often called the principle of 'minimal coupling' as a procedure for how to generalise Special Relativistic quantities to curved spacetime.

Insert 1.2. Finding an inertial frame by freely falling

In order to find an inertial frame, we must find coordinates so that at least locally, at a point x_o^ν, say, we can can do special relativity. This means that we perform a change of coordinates $x^\mu \to x'^\mu(x)$ so that when the metric changes, according to (1.1), the result is

$$g_{\mu\nu}(x_\mathrm{o}^\nu) = \eta_{\mu\nu},$$

where $\eta_{\mu\nu}$ is the Minkowski metric, diag$(-1, +1, \dots,)$. How accurately can we achieve this? In our coordinate transformation, we have in the neighbourhood of x_o^ν:

$$\begin{aligned} x^\mu(x^\nu) &= x^\mu(x_\mathrm{o}^\nu) + \frac{\partial x^\mu}{\partial x'^\nu}(x'^\nu - x_\mathrm{o}'^\nu) \\ &\quad + \frac{1}{2}\frac{\partial^2 x^\mu}{\partial x'^\nu \partial x'^\kappa}(x'^\nu - x_\mathrm{o}'^\nu)(x'^\kappa - x_\mathrm{o}'^\kappa) \\ &\quad + \frac{1}{6}\frac{\partial^3 x^\mu}{\partial x'^\nu \partial x'^\kappa \partial x'^\lambda}(x'^\nu - x_\mathrm{o}'^\nu)(x'^\kappa - x_\mathrm{o}'^\kappa)(x'^\lambda - x_\mathrm{o}'^\lambda)\dots \end{aligned}$$

so we have, at first order, D^2 coefficients to adjust. Since $g'_{\mu\nu}$ has $D(D+1)/2$ components, we are left with

$$D^2 - \frac{D(D+1)}{2} = \frac{D(D-1)}{2}$$

transformations at our disposal. Happily, this is precisely the dimension of the Lorentz group, $SO(D-1,1)$ of rotations and boosts available in our inertial frame. At second order, we have $D^2(D+1)/2$ coefficients to adjust, which is precisely the same number of first derivatives $\partial g'_{\mu\nu}/\partial x'^\kappa$ of the metric that we need to adjust to zero, cancelling all of the 'forces' in the geodesic equation (1.2). At third order, we have $D^2(D+1)(D+2)/6$ coefficients to adjust, while there are $D^2(D+1)^2/4$ second derivatives of the metric, $\partial^2 g'_{\mu\nu}/\partial x'^\kappa \partial x'^\lambda$, to adjust, which is rather more. In fact, this failure to adjust

$$\frac{D^2(D+1)^2}{4} - \frac{D^2(D+1)(D+2)}{6} = \frac{D^2(D^2-1)}{12}$$

second derivatives is of course a statement of physics. This is precisely the number of independent components of the Riemann tensor $R^\lambda_{\kappa\mu\nu}$, which appears in the field equations determining the metric. So everything fits together rather nicely.

A favourite example of a stress tensor for a matter system is the Maxwell system of electromagnetism. Combining the electric potential ϕ and vector potential $\vec{A}$ into a four-vector $\mathbf{A}(\mathbf{x}) = (\phi, \vec{A})$, with components A_μ, the magnetic induction $\vec{B}$ and electric field $\vec{E}$ are captured in the rank two antisymmetric tensor field strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and an observer with four-velocity $\mathbf{u}$ reads the fields as:

$$E_\mu = F_{\mu\nu}u^\nu, \qquad B_\mu = \epsilon_{\mu\nu}{}^{\kappa\lambda}F_{\kappa\lambda}u^\nu, \tag{1.11}$$

where $\epsilon_{\mu\nu\kappa\lambda}$ is the totally antisymmetric tensor in four dimensions, with $\epsilon_{0123} = -1$. (See insert 1.3 for more on this covariant presentation of electromagnetism.) The action is:

$$S_{\rm M} = \int d^D x \mathcal{L} = -\frac{1}{16\pi}\int (-g)^{1/2} F_{\mu\nu}F^{\mu\nu} d^D x, \tag{1.12}$$

and so it is easily verified that the Euler–Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial A_\mu} - \frac{\partial}{\partial x^\nu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)}\right) = 0,$$

give the field equations

$$\nabla_\nu F^{\mu\nu} = 0,$$

where we have used a very useful identity which is easily derived:

$$\delta(-g)^{1/2} = \tfrac{1}{2}(-g)^{1/2} g^{\mu\nu}\delta g_{\mu\nu}. \tag{1.13}$$

On the other hand, since

$$\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} = -\frac{(-g)^{1/2}}{8\pi}\left(g_{\lambda\beta}F^{\mu\lambda}F^{\nu\beta} - \tfrac{1}{4}g^{\mu\nu}F_{\sigma\rho}F^{\sigma\rho}\right) \tag{1.14}$$

the stress tensor is

$$T^{\mu\nu} = \frac{1}{4\pi}\left(g_{\lambda\beta}F^{\mu\lambda}F^{\nu\beta} - \tfrac{1}{4}g^{\mu\nu}F_{\sigma\rho}F^{\sigma\rho}\right). \tag{1.15}$$

1.2 Gravitons and photons

The quantum Field Theorist's most sacred tool is the idea of associating a particle to every sort of field, whether it be matter or force. So a force is

Insert 1.3. Maxwell written covariantly

Probably most familiar is the flat space writing:

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix} \tag{1.16}$$

for the Maxwell tensor. In addition to the four-vector $\mathbf{A}(\mathbf{x}) = (\phi, \vec{A})$, one in general will have a four-current for the source, which combines the current and electric charge density: $\mathbf{J}(\mathbf{x}) = (\rho, \vec{J})$. With these definitions, Maxwell's equations take on a particularly simple covariant form:

$$\nabla_\nu F^{\mu\nu} = -4\pi J^\mu, \qquad \partial_\mu F_{\nu\kappa} + \partial_\nu F_{\kappa\mu} + \partial_\kappa F_{\mu\nu} = 0, \tag{1.17}$$

for the equations with sources, and the source-free equations (Bianchi identity). The energy-momentum tensor for electromagnetism is given in terms of $\mathbf{F}$ in equation (1.15), and is subject to the conservation equation (when the sources $J^\mu = 0$): $\nabla_\mu T^{\mu\nu} = 0$. This contains familiar physics. Specialising to flat space:

$$T_{00} = \frac{1}{8\pi}((\vec{E})^2 + (\vec{B})^2), \qquad T_{0i} = -\frac{1}{4\pi}(\vec{E} \times \vec{B}),$$

which is the familiar expression for the energy density and the momentum density (Poynting vector) of the electromagnetic field

mediated by a particle which propagates along in spacetime between objects carrying the charges of that interaction. There is great temptation to do this for gravity (by allowing all sources of stress-energy-momentum to emit and absorb appropriate quanta), but we immediately run into a conceptual log jam. On the one hand, we have just reminded ourselves of the beautiful picture that gravity is associated to the dynamics of spacetime itself, while on the other hand we would like to think of the gravitational force as mediated by gravitons which propagate on a spacetime background. A technical way of separating out this problem into manageable pieces (up to a point) is to study the linearised theory.

The idea is to treat the metric as split between the background which is say, flat spacetime given by the Minkowski metric $\eta_{\mu\nu}$, diag$(-1, +1, \dots,)$, and some position dependent fluctuation $h_{\mu\nu}(x)$ which is to be small $h_{\mu\nu}(x) \ll 1$. Then the equations determining $h_{\mu\nu}(x)$ are derived from Einstein's equations (1.6) by substituting this ansatz:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x),$$

and keeping only terms linear in $h_{\mu\nu}$.

Let us carry this out. We will raise and lower indices with $\eta_{\mu\nu}$, and note that $g^{\mu\nu}$ will continue to be the inverse metric, which is distinct from $\eta^{\mu\alpha}\eta^{\nu\beta}g_{\alpha\beta}$. Note also that $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$, to the accuracy to which we are working. The affine connection becomes:

$$\Gamma^{\rho}_{\mu\nu} = \tfrac{1}{2}\eta^{\rho\alpha}\left(\partial_\mu h_{\nu\alpha} + \partial_\nu h_{\mu\alpha} - \partial_\alpha h_{\mu\nu}\right), \tag{1.18}$$

and to this order, the Ricci tensor and scalar are:

$$\begin{aligned} R_{\mu\nu} &= \partial^\alpha\partial_{(\nu}h_{\mu)\alpha} - \tfrac{1}{2}\partial^\alpha\partial_\alpha h_{\mu\nu} - \tfrac{1}{2}\partial^\mu\partial_\nu h + O(h^2), \\ R &= \partial^\alpha\partial^\beta h_{\alpha\beta} - \partial^\alpha\partial_\alpha h + O(h^2), \end{aligned} \tag{1.19}$$

where $h = h^\mu_\mu$. Thus we learn that

$$\begin{aligned} R_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}R = \partial^\alpha\partial_{(\nu}h_{\mu)\alpha} - \tfrac{1}{2}\partial^\alpha\partial_\alpha h_{\mu\nu} - \tfrac{1}{2}\partial^\mu\partial_\nu h \\ -\tfrac{1}{2}\eta_{\mu\nu}\left(\partial^\alpha\partial^\beta h_{\alpha\beta} - \partial^\alpha\partial_\alpha h\right) + O(h^2). \end{aligned}$$

Defining $\bar{\gamma}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$, we find our linearised field equations:

$$-\tfrac{1}{2}\partial^\alpha\partial_\alpha \bar{h}_{\mu\nu} + \partial^\alpha\partial_{(\mu}\bar{h}_{\mu)\alpha} - \tfrac{1}{2}\eta_{\mu\nu}\partial^\alpha\partial^\beta\bar{h}_{\beta\gamma} = 8\pi G_{\mathrm{N}}T_{\mu\nu}. \tag{1.20}$$

There is an explicit gauge degree of freedom (recognisable from equation (1.1) as an infinitesimal coordinate transformation)

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu, \tag{1.21}$$

for arbitrary an arbitrary vector ξ_μ. Using this freedom, we *choose the gauge* $\partial^\nu\bar{h}_{\mu\nu} = 0$ (using a gauge transformation satisfying $\partial^\nu\partial_\nu\xi_\mu + \partial^\nu\bar{h}_{\mu\nu} = 0$), which implies

$$\partial^\alpha\partial_\alpha\bar{h}_{\mu\nu} = -16\pi G_{\mathrm{N}}T_{\mu\nu}. \tag{1.22}$$

This is highly suggestive. Consider the system of electromagnetism, with equations of motion (1.17). The equations are invariant under the gauge transformation

$$A_\mu \to A_\mu + \partial_\mu\Lambda,$$

where Λ is an arbitrary scalar. We can use this freedom to choose a gauge $\partial_\mu A^\mu = 0$, (with a parameter satisfying $\partial_\mu\partial^\mu\Lambda + \partial^\nu A_\nu = 0$), which gives the simple equation

$$\partial_\mu\partial^\mu A_\nu = -4\pi J_\nu.$$

This is of a very similar form to what we achieved in equation (1.22) for the system of linearised gravity. The analogy is clear. The Maxwell system has yielded a field equation for a vector (spin one) particle (the photon $A_\mu(x)$) sourced by a vector current ($J_\mu(x)$), while the gravitational system yields the precisely analogous equation for a spin two particle (the graviton $h_{\mu\nu}(x)$) sourced by the stress tensor $T_{\mu\nu}(x)$.

This is the starting point for treating gravity on the same footing as field theory, and in many places later we will have cause to use the word or idea 'graviton', and it is in this sense (a spin two particle propagating on a reference background) that we will mean it. We have seen how to make the delicate journey from the Relativist's geometrical understanding of gravity to a perturbative Field Theorist's. To make the return journey, reconstructing a picture of, say the non-trivial spacetime metric due to a star by starting from the graviton picture is a bit harder, but roughly it is conceptually similar to the same problem in electromagnetism. How does one go from the picture of the photon moving along in spacetime to building up a picture of the strong magnetic fields around a pair of Helmholtz coils? Words and phrases which are offered include 'coherent state of photons', or 'condensation of photons', and these should invoke the idea that the coils' field cannot be constructed using only the perturbative photon picture. One can instead use the photon description to describe processes *in the background* of the Helmholtz field, and we can similarly do the same thing for gravity, describing the propagation of gravitons in the background fields produced by a star. In this way, we see that there is a possibility that there are situations where the conceptual separation between particle quanta and background *in principle* needs be no more dangerous in gravitation than it is in electromagnetism.

Eventually, however, we would like to compute beyond tree level, and the celebrated problems of the theory of gravity treated as a quantum theory will be encountered. Then, the linearised Einstein–Hilbert action

$$S = \frac{1}{16\pi G_{\rm N}} \int d^D x \left(\partial^\alpha\partial^\beta h_{\alpha\beta} - \partial^\alpha\partial_\alpha h\right), \tag{1.23}$$

will eventually reveal itself to be non-renormalisable once we add interactions coming from the next order above linear. In particular, the process of recursively adding counterterms to the bare action in order to define physically measurable quantities does not terminate. As Field Theorists