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Introduction to Dynamical Systems

This book provides a broad introduction to the subject of dynamical systems, suitable for a one- or two-semester graduate course. In the first chapter, the authors introduce over a dozen examples, and then use these examples throughout the book to motivate and clarify the development of the theory. Topics include topological dynamics, symbolic dynamics, ergodic theory, hyperbolic dynamics, one-dimensional dynamics, complex dynamics, and measure-theoretic entropy. The authors top off the presentation with some beautiful and remarkable applications of dynamical systems to such areas as number theory, data storage, and Internet search engines.

This book grew out of lecture notes from the graduate dynamical systems course at the University of Maryland, College Park, and reflects not only the tastes of the authors, but also to some extent the collective opinion of the Dynamics Group at the University of Maryland, which includes experts in virtually every major area of dynamical systems.

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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
 The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
 The Edinburgh Building, Cambridge CB2 2RU, UK
 40 West 20th Street, New York, NY 10011-4211, USA
 477 Williamstown Road, Port Melbourne, VIC 3207, Australia
 Ruiz de Alarcón 13, 28014 Madrid, Spain
 Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

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First published 2002

Printed in the United Kingdom at the University Press, Cambridge

Typefaces Times Ten 10/13 pt. and Avenir *System* L^AT_EX 2_ε [TB]

A catalog record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data

Brin, Michael.

Introduction to dynamical systems / Michael Brin, Garrett Stuck.

p. cm.

Includes bibliographical references and index.

ISBN 0-521-80841-3

1. Differentiable dynamical systems. I. Stuck, Garrett, 1961– II. Title.

QA614.8 .B75 2002

514'.74 – dc21

2002022281

ISBN 0 521 80841 3 hardback

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To Eugenia, Pamela, Sergey, Sam, Jonathan, and Catherine
for their patience and support.

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Introduction

The purpose of this book is to provide a broad and general introduction to the subject of dynamical systems, suitable for a one- or two-semester graduate course. We introduce the principal themes of dynamical systems both through examples and by explaining and proving fundamental and accessible results. We make no attempt to be exhaustive in our treatment of any particular area.

This book grew out of lecture notes from the graduate dynamical systems course at the University of Maryland, College Park. The choice of topics reflects not only the tastes of the authors, but also to a large extent the collective opinion of the Dynamics Group at the University of Maryland, which includes experts in virtually every major area of dynamical systems.

Early versions of this book have been used by several instructors at Maryland, the University of Bonn, and Pennsylvania State University. Experience shows that with minor omissions the first five chapters of the book can be covered in a one-semester course. Instructors who wish to cover a different set of topics may safely omit some of the sections at the end of Chapter 1, §§2.7–2.8, §§3.5–3.8, and §§4.8–4.12, and then choose from topics in later chapters. Examples from Chapter 1 are used throughout the book. Chapter 6 depends on Chapter 5, but the other chapters are essentially independent. Every section ends with exercises (starred exercises are the most difficult).

The exposition of most of the concepts and results in this book has been refined over the years by various authors. Since most of these ideas have appeared so often and in so many variants in the literature, we have not attempted to identify the original sources. In many cases, we followed the written exposition from specific sources listed in the bibliography. These sources cover particular topics in greater depth than we do here, and we recommend them for further reading. We also benefited from the advice and guidance of a number of specialists, including Joe Auslander, Werner Ballmann,

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Introduction

Ken Berg, Mike Boyle, Boris Hasselblatt, Michael Jakobson, Anatole Katok, Michal Misiurewicz, and Dan Rudolph. We thank them for their contributions. We are especially grateful to Vitaly Bergelson for his contributions to the treatment of applications of topological dynamics and ergodic theory to combinatorial number theory. We thank the students who used early versions of this book in our classes, and who found many typos, errors, and omissions.