Lévy Statistics and Laser Cooling

How Rare Events Bring Atoms to Rest

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1
Introduction

This book deals with the important developments that have recently occurred in two different research fields, laser manipulation of atoms on the one hand, non-Gaussian statistics and anomalous diffusion processes on the other hand. It turns out that fruitful exchanges of ideas and concepts have taken place between these two apparently disconnected fields. This has led to cross-fertilization of each of them, providing new physical insights into the most efficient laser cooling mechanisms as well as simple and mathematically soluble examples of anomalous random walks.

We thought that it would be useful to present in this book a detailed report[1] of these developments. Our ambition is to try to improve the dialogue between different communities of scientists and, hopefully, to stimulate new, interesting developments. This book is therefore written as a case study accessible to the non-specialist.

Our aim is also to promote, within the atomic physics and quantum optics community, a way to approach and solve problems that is less based on exact solutions, but relies more on the identification of the physically relevant features, thus allowing one to construct simplified, idealized models and qualitative (and sometimes quantitative) solutions. This approach is of course common in statistical physics, where, often, details do not matter, and only robust global features determine the relevant physical properties. Laser cooling is an ideal case study, where the power of this methodology is clearly illustrated.

1.1 Laser cooling

During the last two decades, atomic physics has undergone spectacular progress based on a new experimental method, called laser cooling and trapping. By using

[1] Only a preliminary brief report of this work has been published [BBE94]. More detailed versions have been presented in an unpublished thesis work [Bar95] and in lecture notes [Coh96].

resonant or quasi-resonant exchanges of energy and momentum between atoms and laser light, it is now possible to obtain samples of atoms at temperatures in the microkelvin and even in the nanokelvin range, i.e. with velocities in the cm/s or in the mm/s range [Chu98, Coh98, Phi98]. Further cooling and increase of the density in phase space can then be achieved by using another recently developed method, called evaporative cooling. This has opened the way to a wealth of new investigations, ranging from ultrahigh resolution spectroscopy and atomic clocks to atomic interferometry and Bose–Einstein condensation (for a review of these fields see, for example, [AdR97, APS92, MeV99, BEC]).

In standard laser cooled atomic samples, called ‘optical molasses’, the ensemble of atoms interacting with appropriate sets of laser beams reaches a steady-state resulting from competition between two effects: damping of the atomic momenta due to a *friction* force originating from various types of velocity-dependent mechanisms (‘Doppler’ or ‘Sisyphus’ cooling) on the one hand and increase of the atomic momenta, or *momentum diffusion*, due to the fluctuations of the radiative forces, on the other. These fluctuations are associated with the random atomic recoils occurring in spontaneous emission processes which are generally unavoidable in any cooling scheme and which make the evolution of the atomic momentum look like a *random walk*. For a single spontaneous emission event, the recoil momentum of the atom has a magnitude (single photon recoil)

$$p_{\mathrm{R}} = \hbar k, \tag{1.1}$$

where $\hbar k$ is the momentum of a photon (k is the optical wave-vector). It is therefore not surprising that, usually, the steady-state *rms* atomic momentum δp cannot be smaller than p_{R}: this is the so-called *single photon recoil limit* of laser cooling.

1.2 Subrecoil laser cooling

A completely different approach to laser cooling can be followed, which is not based on a friction force and where the single photon recoil no longer appears as a fundamental limit. The basic idea, presented in Fig. 1.1, is to create a ‘trap’ in momentum space, consisting of a small volume around $\mathbf{p} = \mathbf{0}$ ($\mathbf{p}$ denotes the atomic momentum), which the atoms can reach during their random walk and where they stay for a very long time, which increases indefinitely when $\mathbf{p} \to \mathbf{0}$. Such a situation is achieved by making the photon scattering rate (fluorescence rate) $R(\mathbf{p})$ vanish when $\mathbf{p} \to \mathbf{0}$. The random walk in momentum space slows down when $\mathbf{p}$ decreases and stops when $\mathbf{p} = \mathbf{0}$, so that atoms remain stuck in the neighbourhood of $\mathbf{p} = \mathbf{0}$. Up to now, this has been demonstrated by two methods, Velocity Selective Coherent Population Trapping (VSCPT) [AAK88] and Raman cooling [KaC92].

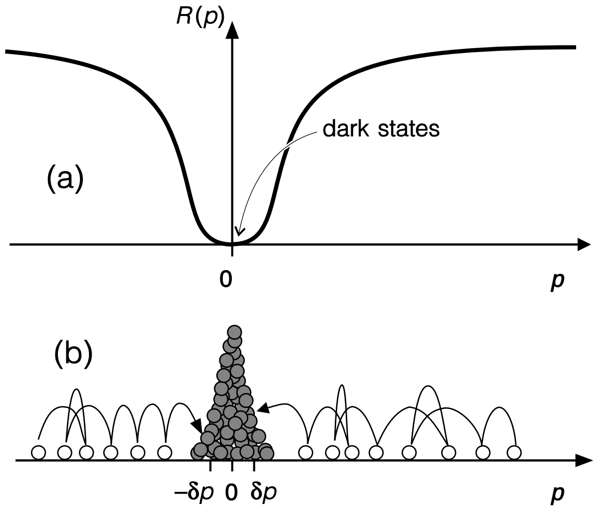

Fig. 1.1. Principle of subrecoil cooling. (a) The fluorescence rate $R(p)$ vanishes at momentum $p = 0$. (b) The atoms perform a random walk in p-space and accumulate in the vicinity of $p = 0$.

Cooling, i.e. an increase of the momentum space density in a narrow range around $\mathbf{p} = \mathbf{0}$, no longer results here from a friction force pushing the atoms towards $\mathbf{p} = \mathbf{0}$, but from a combination of two effects: momentum diffusion and vanishing of the jump rate of the random walk when $\mathbf{p} \to \mathbf{0}$. Another important difference between this and other cooling schemes is the absence of a steady-state value of the momentum distribution: the longer the interaction time θ, the narrower the range δp around $\mathbf{p} = \mathbf{0}$ in which the atoms can remain trapped during θ. Because of the absence of a steady-state and because of the existence of atomic characteristic times (trapping times) that can be longer than the observation time, here we will call such cooling *non-ergodic cooling*. It has also been called *subrecoil cooling* because nothing now prevents the atomic momentum spread δp reaching values smaller than the photon momentum $\hbar k$.

1.3 Subrecoil cooling and Lévy statistics

We present in this book a new general description of non-ergodic or subrecoil cooling in terms of a competition between trapping processes (i.e. the atom falls in the trap) and recycling processes (i.e. the atom leaves the trap and eventually returns to it). The fundamental feature which has stimulated the new approach presented in this book is that the distributions of trapping times and escape times can be very broad, so broad that the usual *Central Limit Theorem* (CLT) can no longer be applied to study the distributions of the total trapping time and of the total recycling time after N entries in the trap separated by N exits. We show in this book that the so-called *Lévy statistics,* which generalizes the CLT to broad distributions with power-law tails, is the appropriate tool for this problem and that it can provide quantitative results for all the important characteristics of cooled atoms.

Lévy statistics is an outcome of some fundamental mathematical work performed in the 1930's [Lev37, GnK54]. The goal was then to find stable distribution laws for the sum of N independent random variables, i.e. the distribution laws that keep the same mathematical form when $N \to \infty$. Gaussian distributions and Lévy distributions are the solutions of this problem. While the immense applicability of Gaussian distributions was recognized long ago, Lévy laws have been unduly ignored in the natural sciences and have been considered a sheer mathematical curiosity. However, the situation has completely changed over the last 15 years. Lévy statistics is now recognized as the best tool for studying many anomalous diffusion problems for which standard statistics are inadequate. Application fields include not only physics (anomalous diffusion, chaotic dynamics, mechanics of sandpiles, . . .) but also finance, biology, etc. (see [BoG90, SZF95, Bak96, BCK97, Man97, Zas99, MaS99, PaB99, BoP00, CoR00, GoL01]). Lévy statistics can handle situations in which the standard deviation (or even the average value) of the studied random variable does not exist. It provides technical tools for performing calculations. Importantly, Lévy statistics implies properties that depart very strongly, not only quantitatively but even qualitatively, from usual statistical behaviour. For instance, when the average value of a random variable x is infinite, the sum $\sum_{i=1}^{N} x_i$ is no longer proportional to the number N of terms (usual law of large numbers), but a different *scaling* behaviour is obtained. This of course has dramatic phenomenological consequences, as we will see for the specific case of subrecoil laser cooling.

From the point of view of laser cooling, the study of subrecoil cooling by Lévy statistics turns out to be extremely fruitful. First, it allows one to extract the key ingredients of the cooling process from the relatively complicated microscopic description of the problem provided by atomic physics. Moreover, the statistical approach leads to unique analytical predictions for the asymptotic properties of the cooled atoms, independent of the details of the particular cooling scheme considered, as expected when one goes from a microscopic description to a statistical description.

From the point of view of statistics, this work can also be considered as a case study for the application of Lévy statistics in a privileged situation where the statistical model can be derived from first principles, developed analytically and, finally, precisely compared to microscopic theoretical approaches and to experimental results.

1.4 Content of the book

This book is intended for two different communities working in two different fields: atomic physics and quantum optics on the one hand and statistical processes on the other. We have thus considered it useful to include a summary of important results already known to each community, but not necessarily by both. These basic results are presented in Chapter 2 (laser cooling, see also Appendix A) and Chapter 4 (Lévy statistics), while Chapter 3 introduces the models that connect both fields. We then proceed in Chapters 5 and 6 with the derivation of the central results of this work. These results are then interpreted, discussed and extended in Chapters 7, 8 and 9. Appendices present several technical developments, either on Lévy statistics, or on subrecoil cooling processes.

More precisely, in *Chapter 2*, we recall some atomic physics results on laser cooling and subrecoil cooling. We point out the difficulties of an exact quantum treatment of subrecoil cooling using the generalized optical Bloch equations, and we present a more efficient quantum approach based on stochastic descriptions of the evolution of the wave function of the system, in terms of quantum jumps occurring at random times. This approach provides Monte Carlo simulations of the quantum evolution of the atomic momentum which allow one to describe, in a rigorous way, the cooling process by inhomogeneous random walks in momentum space with a momentum-dependent jump rate $R(p)$ vanishing for $p = 0$.

Such an approach suggests a simplified model where we make a partition between two classes of atoms: (i) the cold atoms, which are in a trapping volume in momentum space where the momentum $\mathbf{p}$ is close to zero, and which stay for a long time τ (trapping time) in this trapping volume (trapped atoms); (ii) atoms outside of the trapping volume, which make a random walk of duration $\hat{\tau}$ in momentum space, under the effect of radiation, until they come back again in the trapping volume ($\hat{\tau}$ is a first return time). We calculate in *Chapter 3* the probability distributions $P(\tau)$ and $\hat{P}(\hat{\tau})$ of the trapping times and first return times, and we show that in several important cases these distributions are broad distributions with power-law tails, for which Lévy statistics provides the relevant statistical treatment.

Chapter 4 summarizes the main results of Lévy statistics needed for the derivation and the interpretation of the results presented in this work. We will not give here all the detailed proofs, but rather emphasize the physical meaning of the results and the important differences between Lévy statistics and usual Gaussian statistics. More details may be found in [GnK54, Lev37, BoG90]. We also introduce a ‘sprinkling distribution’ which will be the basic tool for the calculations of the following chapters.

The concepts introduced in Chapter 4 are used in the following chapters for the derivation of quantitative predictions concerning laser cooling. We first study, in

Chapter 5, the proportion $f_{\text{trap}}(\theta)$ of cooled atoms after a cooling time θ, for the various cases considered in Chapter 3. We find not only the asymptotic behaviour of $f_{\text{trap}}(\theta)$, but the rate at which this asymptotic behaviour is reached when $\theta \longrightarrow \infty$. This allows us to give a first characterization of the efficiency of the cooling process. An important result of this chapter is also that $f_{\text{trap}}(\theta)$, defined here as an ensemble average, can be different from the corresponding time average. This clearly shows the *non-ergodic* character of the laser cooling process considered here.

A further step is achieved in *Chapter 6* by calculating the momentum distribution $\mathcal{P}(\mathbf{p})$ of the cold (trapped) atoms and the momentum distribution $\pi(p)$ along a given axis. We show that there is always in $\pi(p)$ a narrow peak whose width δp tends to zero when $\theta \longrightarrow \infty$, and the fraction of atoms contained in this peak is calculated in several important cases. The tails of $\pi(p)$ at large p are also studied. Their decrease is described by a power law, which shows that $\pi(p)$ is not a Gaussian distribution so that it is not possible, strictly speaking, to define a thermodynamic temperature. Finally, the increase of the density of atoms in momentum space and in phase space is evaluated for the various situations considered in this work.

The physical content of the results obtained in the preceding chapters is discussed in detail in *Chapter 7*. We re-interpret them in terms of rate equations describing a competition between a rate of entry in the trapping volume and a rate a departure. The rate of entry $S_{\text{R}}(t)$ is in fact nothing but the 'sprinkling distribution' introduced in Chapter 4. This enables one to interpret the behaviour of the height and of the peak of the momentum distribution. We also discuss in Chapter 7 a few other problems: the effect of a non-vanishing jump rate when $p \to 0$ and the connection between non-stationarity, non-ergodicity and broad distributions.

In *Chapter 8* we compare the analytical predictions of the statistical approach presented in this book with experimental results, as well as with predictions of microscopic theoretical approaches based on microscopic quantum treatments (stochastic wave function simulations or generalized optical Bloch equations). The excellent agreement between the various results gives us confidence in the approach developed in this work and in the approximations upon which it is based.

In *Chapter 9* we present an example of application of the approach developed in this book to a specific problem: the optimization of the height of the peak of cooled atoms. This brings into play both the insights and the technical results obtained in previous chapters and deepens our understanding of some properties of non-ergodic cooling.

We finally summarize in *Chapter 10* the main results derived in this book. We also mention a few possible extensions and a few open problems.