Index of main notation

Latin symbols

Notation	Definition	Page
$\mathcal{A}_\mu$	prefactor of $P(\tau)$ depending on the type of distribution for $P(\tau\|p)$	33
C_D	volume of the unit sphere in D dimensions: $C_1 = 2,\ C_2 = \pi,\ C_3 = 4\pi/3$	32
$\mathcal{D}(\theta)$	phase space density in $\mathbf{p} = \mathbf{0}$ at time θ	74
$f_{\text{peak}}(\theta)$	fraction of cooled atoms ($p < p_\theta$) at time θ	74
$f_{\text{trap}}(\theta)$	proportion of trapped atoms ($p < p_{\text{trap}}$) at time θ	60, 71
$f_{\text{trap}}(\theta)\big\|_{\text{opt}}$	proportion of trapped atoms evaluated at the time θ for which $h(\theta)$ is optimized	134
F	multiplicative factor allowing VSCPT quantum calculations to give the unconfined model	153
$\mathcal{F}_p$	family of the three states $\left\{\|g_-\rangle_p, \|g_+\rangle_p, \|e\rangle_p\right\}$ coupled by absorption and stimulated emission in one-dimensional σ_+/σ_- VSCPT	146
$\mathcal{G}(q)$	rescaled momentum distribution: $\pi(p,\theta) = h(\theta)\,\mathcal{G}(p/p_\theta)$. Case $\langle\tau\rangle$ infinite and $\langle\hat{\tau}\rangle$ finite	76
$\widetilde{\mathcal{G}}(q)$	same definition as $\mathcal{G}(q)$ but with $\langle\tau\rangle$ and $\langle\hat{\tau}\rangle$ finite	81
$\hat{\mathcal{G}}(q)$	same definition as $\mathcal{G}(q)$ but with $\langle\tau\rangle$ and $\langle\hat{\tau}\rangle$ infinite	85
$h(\theta)$	height of the peak of cooled atoms: $h(\theta) = \mathcal{P}(\mathbf{p} = \mathbf{0}, \theta)$	73

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More information

Notation	Definition	Page
$h(\theta)\|_{\text{opt}}$	optimized height of the peak of cooled atoms	134
$\hbar k$	photon momentum	2
$\hat{H}_p$	effective Hamiltonian describing the reduced evolution within $\mathcal{F}_p$	148
k	photon wave-vector modulus	2
k_{B}	Boltzmann constant	8
$\mathcal{L}f(s)$	Laplace transform of any function $f(t)$	45
$L_\mu(\xi)$	completely asymmetric Lévy distribution of index μ	45
M	atomic mass	8
N	number of terms in a sum $T_N = \sum_{i=1}^N \tau_i$; in particular,	44
	number of trapping events during the interaction time θ	4, 23
N_{samp}	number of samples (atoms) used in quantum jump simulations	108
$\mathbf{p}$	atomic momentum	2
p	modulus of the atomic momentum: $p = \\|\mathbf{p}\\|$	25, 71
$p_{1/2}$	half-width of function $\tau(p)$ when $R_0 \neq 0$: $R_0 = \frac{1}{\tau_0}\left(\frac{p_{1/2}}{p_0}\right)^\alpha$	95
p_{D}	Doppler width: $kp_{\text{D}}/M = \Gamma/2$	26
$p_{\text{m}}(\theta)$	median momentum of the trapped atoms at time θ	73
p_0	width of the dip of the jump rate $R(p)$ in $p \simeq 0$	23, 25
p_{max}	wall in momentum space (cf. effect of friction forces)	25
p_{R}	single photon recoil: $p_{\text{R}} = \hbar k$	2
p_{trap}	size of the momentum trap	23
p_θ	characteristic momentum at time θ: $R(p_\theta)\cdot\theta = 1$	70
$p_{\theta,\text{opt}}$	characteristic momentum p_θ evaluated at the time θ for which $h(\theta)$ is optimized	133
$P(x)$	probability distribution (probability density) of any variable x	29

Notation	Definition	Page
$P_i(p_x)$	jump rate induced by the i^{th} pulse in Raman cooling	164–166
$P(\tau)$	probability distribution of trapping times τ, for an atom landing anywhere in the trap	5, 24
$P(\tau\|p)$	probability distribution of the sojourn times $\tau(p)$ at momentum p (deterministic or exponential model)	30, 71
$\hat{P}(\hat{\tau})$	probability distribution of recycling times $\hat{\tau}$	5, 24
$\mathcal{P}(\mathbf{p})$	probability distribution of the atomic momentum vector $\mathbf{p}$	69, 72
$\mathcal{P}(p,\theta)$	probability distribution of the momentum modulus p at time θ	71
Q	atomic parameter determined by the specific laser configuration and appearing in the expression of $\tau_{\rm b}$	126, 127
$\hat{Q}$	atomic parameter determined by the specific laser configuration and appearing in the expression of $\langle\hat{\tau}\rangle$	127
$R(\mathbf{p})$	jump rate (i.e. fluorescence rate, photon scattering rate): $R(p) = 1/\langle\tau(p)\rangle$	2, 20
R_0	jump rate in $p = 0$: $R_0 = R(p = 0)$	26, 93
s	conjugate of a time variable through a Laplace transform	39, 46
$S(t)$	'sprinkling distribution' (renewal density)	55, 56
S_D	surface of unit sphere in D dimensions: $S_D = DC_D$	32
$S_{\rm E}(t)$	sprinkling distribution of exit times, i.e. rate of exit from the trap	62
$S_{\rm R}(t)$	sprinkling distribution of return times, i.e. rate of entry in the trap	62
t_l	last trapping time before $t = \theta$	62
T	effective temperature: $k_{\rm B}T = \delta p^2/M$	8
T_N	sum of any N independent positive random variables τ_i (in Chapter 4): $T_N = \sum_{i=1}^{N}\tau_i$;	44
	in particular, total trapping time during an interaction of duration θ (in other chapters)	24

Notation	Definition	Page
$\hat{T}_N$	total recycling time during an interaction of duration θ: $\hat{T}_N = \sum_{i=1}^{N} \hat{\tau}_i$	24
T_{R}	recoil temperature: $T_{\mathrm{R}} = \hbar^2 k^2/(Mk_{\mathrm{B}})$	9
$V_D(p)$	volume of a sphere of radius p in D dimensions: $V_D(p) = C_D\, p^D$	32
$w(\theta)$	half-width at $\mathrm{e}^{-1/2}$ of the peak of cooled atoms (idem δp): $\pi(p = w(\theta), \theta) = \mathrm{e}^{-1/2}\, \pi(p = 0, \theta)$	70, 73
$W(\tau)$	waiting time distribution (delay function), i.e. distribution of the time intervals τ between two *successive* spontaneous emissions	14, 58
$Y(x)$	Heaviside function ($x < 0$: $Y(x) = 0$; $x \geq 0$: $Y(x) = 1$)	49, 72

Greek symbols

Notation	Definition	Page
α	exponent of p ($< p_0$) in $R(p)$: $R(p) = \frac{1}{\tau_0}\left(\frac{p}{p_0}\right)^{\alpha}$	25
$\gamma(\beta, x)$	incomplete Gamma function: $\gamma(\beta, x) = \int_0^x e^{-u} u^{\beta-1} du$	31
Γ	natural width of the excited state e. Γ^{-1} is the radiative lifetime of e.	148
$\Gamma_{g_\pm}(p)$	decay rates of states $\lvert g_\pm\rangle_p$	152
Γ_j	decay rates of the three eigenmodes of $\hat{H}_p$: $\Gamma_3(p) \simeq \Gamma$, $\Gamma_1(p) = \Gamma_C(p)$, $\Gamma_2(p) = \Gamma_{NC}(p)$	148–152
$\Gamma(x)$	gamma function: $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$	31, 59
$\tilde{\delta}$	effective detuning between the laser frequency ω_L and the atomic frequency ω_A, including the recoil frequency Ω_R: $\tilde{\delta} = \omega_L - \omega_A + \Omega_R$	148
δp	half-width at $e^{-1/2}$ of the peak of cooled atoms (idem $w(\theta)$)	8
Δp	root mean square (rms) step length of the momentum random walk	22, 28
θ	duration of the interaction between the atoms and the laser beams	3, 12
λ	optimization parameter appearing in the expressions of τ_b and $\langle\hat{\tau}\rangle$: $\lambda = (\Omega_1/\Gamma)^2$ for VSCPT, $\lambda = 1/(\Gamma\tau_{p,1})$ for Raman cooling	126–127
λ_A	wavelength associated with the atomic transition with frequency ω_A: $\lambda_A = 2\pi c/\omega_A$	148
$\lambda_{opt}(\theta)$	value of λ optimizing the height $h(\theta)$ of the momentum distribution after an interaction time θ	131
μ	exponent of a power-law probability distribution $P(\tau) \underset{\tau\to\infty}{\simeq} \mu\tau_b^{\mu}/(\tau^{1+\mu})$; in particular, exponent of the probability	43
	distribution of trapping times: $\mu = D/\alpha$	33
$\hat{\mu}$	exponent of the probability distribution of recycling times (when power-law distributed)	41
$\pi(p, \theta)$	reduced momentum distribution, more precisely one-dimensional section of the three-dimensional momentum distribution	72

Notation	Definition	Page
$\Pi_N(T_N)$	probability distribution of a sum T_N	46
$\rho(p)$	probability density for an atom entering the trap to land at a momentum of modulus p	32
$\rho(p_x)$	probability density for an atom entering the trap to land at momentum p_x	29
τ	any positive random variable distributed as a power law for large τ	43
	in particular: trapping time, i.e. (random) sojourn time in the trap	5, 23
$\hat{\tau}$	recycling time, i.e. first return time	5, 23
τ_0	inverse of the jump rate $R(p)$ at saturation	26
τ_{b}	scale of random variable τ distributed as a power law for large τ: $P(\tau) \underset{\tau\to\infty}{\simeq} \mu\tau_{\mathrm{b}}^{\mu}/(\tau^{1+\mu})$	43
	in particular: scale of trapping times	33
$\hat{\tau}_{\mathrm{b}}$	scale of recycling times (when power-law distributed)	41
$\tau(p)$	sojourn time at momentum p: $\langle\tau(p)\rangle = 1/R(p)$	12
$\tau_{\mathrm{p},i}$	duration of the i^{th} pulse used in Raman cooling with square pulses	164, 165
$\tau_{\mathrm{ret}}(\theta)\|_{\mathrm{opt}}$	average return time to the peak of half-width $p_{\theta,\,\mathrm{opt}}$ for the optimum condition $\lambda = \lambda_{\mathrm{opt}}(\theta)$	135
τ_{trap}	average sojourn time at $p = p_{\mathrm{trap}}$: $\tau_{\mathrm{trap}} = 1/R(p_{\mathrm{trap}})$	29
$\|\Psi_{\mathrm{NC}}(p)\rangle$	non-coupled state of $\hat{H}_p$, associated with the decay rate $\Gamma_2(p)$	151
$\|\Psi_{\mathrm{C}}(p)\rangle$	coupled state of $\hat{H}_p$, associated with the decay rate $\Gamma_1(p)$	151
$\psi(\tau\|p)$	probability that the trapping time exceeds τ for an atom with momentum p: $\psi(\tau\|p) = \int_\tau^\infty \mathrm{d}\tau' P(\tau'\|p)$	71
Ω_1	Rabi frequency (defined for a transition with a Clebsch–Gordan coefficient equal to one)	148
$\Omega_{1,\,\mathrm{opt}}$	Rabi frequency optimized for an experiment of duration θ	133
Ω_{R}	angular frequency associated with the recoil kinetic energy $\hbar^2k^2/2M$: $\Omega_{\mathrm{R}} = \hbar k^2/2M$	148

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