Lévy Statistics and Laser Cooling

How Rare Events Bring Atoms to Rest

Laser cooling of atoms provides an ideal case study for the application of Lévy statistics in a privileged situation where statistical models can be derived from first principles. This book establishes profitable connections between these two research fields, and demonstrates how the most efficient laser cooling techniques can be simply and quantitatively understood in terms of non-ergodic random processes dominated by a few rare events.

Lévy statistics is now recognized as the proper framework for analysing many different problems (in physics, biology, earth sciences, finance, etc.) for which standard Gaussian statistics is inadequate. Lévy statistics involves random variables with such broad distributions that the usual Central Limit Theorem no longer holds. Laser cooling allows atoms to be cooled to very low temperatures and brought to rest, and is a new research field with many applications. It provides a fruitful example of how approaches based on Lévy statistics can yield analytic predictions that can then be compared with both microscopic quantum optics treatments and experimental results.

The authors of this book are world leaders in the fields of laser cooling, light–atom interactions and statistical physics, and are also renowned for their clarity of exposition. Since the subject of this book embraces several different research areas, the authors have made every effort to ensure that it remains comprehensible to the non-specialist. They explain the important concepts of laser cooling and give an introduction to the concept of random walks and Lévy statistics, such that no detailed prior knowledge is required. This book will therefore be of great interest to researchers in the fields of atomic physics, quantum optics and statistical physics, as well as to engineers and mathematicians interested in stochastic processes. It will also be most useful for illustrating graduate courses on these topics.

FRANÇOIS BARDOU is a researcher at the Centre National de la Recherche Scientifique (CNRS) and currently works on problems in quantum stochastics at the Institut de Physique et de Chimie des Matériaux de Strasbourg. He received the 1995 Aimé Cotton prize (Atomic Physics prize of the French Physical Society) for his experimental and theoretical studies of laser cooling performed at the École Normale Supérieure de Paris. These studies, in collaboration with his co-authors, provided the basis for this book.

JEAN-PHILIPPE BOUCHAUD is a Senior Expert at the Service de Physique de l'État Condensé and at CEA-Saclay. In 1994 he founded his own company, 'Science and Finance', and continues to have diverse research interests which include statistical physics, granular matter and theoretical finance. In 1996 he won the

CNRS Silver Medal. He is in charge of a number of statistical physics and finance courses in various Grandes Écoles, Paris, and is the co-author of *Theory of Financial Risk* (Cambridge University Press, 2000).

ALAIN ASPECT is a Director of Research at CNRS and a Professor at the École Polytechnique, Palaiseau. After completing, in the early 1980s, a series of experiments on the foundations of quantum mechanics, he joined Claude Cohen-Tannoudji at the École Normale Supérieure to work on laser cooling of atoms. He is now head of the Atom Optics group of Institut d'Optique at Orsay and is the co-author of *Introduction to Lasers and Quantum Optics* (Cambridge University Press, in preparation).

CLAUDE COHEN-TANNOUDJI is Professor of Atomic and Molecular Physics at the Collège de France in Paris and was honoured with the Nobel Prize for Physics in 1997 for his work on the development of methods to cool and trap atoms with laser light. He is also the co-author of three other books: *Quantum Mechanics* (1992), *Photons and Atoms: Introduction to Quantum Electrodynamics* (1989), and *Atom–Photon Interactions: Basic Processes and Applications* (1998).

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Foreword

Long ago, Paul Lévy invented a strange family of random walks – where each segment has a very broad probability distribution. These flights, when they are observed on a macroscopic scale, do not follow the standard Gaussian statistics. When I was a student, Lévy's idea appeared to me as (a) amusing, (b) simple – all the statistics can be handled via Fourier transforms – and (c) somewhat baroque: where would it apply?

As often happens with new mathematical ideas, the fruits came later. For example, É. Bouchaud proved that adsorbed polymer chains often behave like Lévy flights. In a very different sector, J.P. Bouchaud showed the role of Lévy distributions in risk evaluation. Now we meet a third major example, which is described in this book: cold atoms.

The starting point is a jewel of quantum physics: we think of an atom in a state of 0 translational momentum $p = 0$ (zero Doppler effect), inside a suitably prescribed laser field. For instance, with an angular momentum $J = 1$ we can have two ground states $|+\rangle$ and $|-\rangle$, and one excited state $|0\rangle$. The particular state $|+\rangle + |-\rangle$ has an admirable property: it is entirely decoupled from the radiation and can live for an indefinitely long time. It is thus possible to create a trap (around $p = 0$ in momentum space) in which the atoms will live for very long times: this so-called 'subrecoil laser cooling' has been a major advance of recent years. There are many statistical questions, concerning the resulting random flights in momentum space with alternate sequences of trapping and recycling. All the resulting effects in p space and in the time sequence can be measured and compared with statistical predictions inspired by the Lévy flights. (Here, the broad distributions are in the lifetimes, not in the size of the jumps.)

The present book summarizes these advances, incorporating a rare admixture of quantum physics and classical statistics. It is a meeting point for two cultures, each of them being represented by outstanding experts.

I am very impressed by this combination and by the clarity of the result. Both atomic physics and statistical physics integrate (roughly) a hundred years of culture. To extract what is needed from the two cultures and to make it accessible to a simple physicist was a real challenge. This joint group has done it. I am sure that many scientists will feel a special pleasure when reading the book – and that it will last a long time.

P.G. de Gennes
February 2001

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