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Introduction

This book deals with the important developments that have recently occurred in two different research fields, laser manipulation of atoms on the one hand, non-Gaussian statistics and anomalous diffusion processes on the other hand. It turns out that fruitful exchanges of ideas and concepts have taken place between these two apparently disconnected fields. This has led to cross-fertilization of each of them, providing new physical insights into the most efficient laser cooling mechanisms as well as simple and mathematically soluble examples of anomalous random walks.

We thought that it would be useful to present in this book a detailed report¹ of these developments. Our ambition is to try to improve the dialogue between different communities of scientists and, hopefully, to stimulate new, interesting developments. This book is therefore written as a case study accessible to the non-specialist.

Our aim is also to promote, within the atomic physics and quantum optics community, a way to approach and solve problems that is less based on exact solutions, but relies more on the identification of the physically relevant features, thus allowing one to construct simplified, idealized models and qualitative (and sometimes quantitative) solutions. This approach is of course common in statistical physics, where, often, details do not matter, and only robust global features determine the relevant physical properties. Laser cooling is an ideal case study, where the power of this methodology is clearly illustrated.

1.1 Laser cooling

During the last two decades, atomic physics has undergone spectacular progress based on a new experimental method, called laser cooling and trapping. By using

¹ Only a preliminary brief report of this work has been published [BBE94]. More detailed versions have been presented in an unpublished thesis work [Bar95] and in lecture notes [Coh96].

resonant or quasi-resonant exchanges of energy and momentum between atoms and laser light, it is now possible to obtain samples of atoms at temperatures in the microkelvin and even in the nanokelvin range, i.e. with velocities in the cm/s or in the mm/s range [Chu98, Coh98, Phi98]. Further cooling and increase of the density in phase space can then be achieved by using another recently developed method, called evaporative cooling. This has opened the way to a wealth of new investigations, ranging from ultrahigh resolution spectroscopy and atomic clocks to atomic interferometry and Bose–Einstein condensation (for a review of these fields see, for example, [AdR97, APS92, MeV99, BEC]).

In standard laser cooled atomic samples, called ‘optical molasses’, the ensemble of atoms interacting with appropriate sets of laser beams reaches a steady-state resulting from competition between two effects: damping of the atomic momenta due to a *friction* force originating from various types of velocity-dependent mechanisms (‘Doppler’ or ‘Sisyphus’ cooling) on the one hand and increase of the atomic momenta, or *momentum diffusion*, due to the fluctuations of the radiative forces, on the other. These fluctuations are associated with the random atomic recoils occurring in spontaneous emission processes which are generally unavoidable in any cooling scheme and which make the evolution of the atomic momentum look like a *random walk*. For a single spontaneous emission event, the recoil momentum of the atom has a magnitude (single photon recoil)

$$p_R = \hbar k, \tag{1.1}$$

where $\hbar k$ is the momentum of a photon (k is the optical wave-vector). It is therefore not surprising that, usually, the steady-state *rms* atomic momentum δp cannot be smaller than p_R : this is the so-called *single photon recoil limit* of laser cooling.

1.2 Subrecoil laser cooling

A completely different approach to laser cooling can be followed, which is not based on a friction force and where the single photon recoil no longer appears as a fundamental limit. The basic idea, presented in Fig. 1.1, is to create a ‘trap’ in momentum space, consisting of a small volume around $\mathbf{p} = \mathbf{0}$ ($\mathbf{p}$ denotes the atomic momentum), which the atoms can reach during their random walk and where they stay for a very long time, which increases indefinitely when $\mathbf{p} \rightarrow \mathbf{0}$. Such a situation is achieved by making the photon scattering rate (fluorescence rate) $R(\mathbf{p})$ vanish when $\mathbf{p} \rightarrow \mathbf{0}$. The random walk in momentum space slows down when $\mathbf{p}$ decreases and stops when $\mathbf{p} = \mathbf{0}$, so that atoms remain stuck in the neighbourhood of $\mathbf{p} = \mathbf{0}$. Up to now, this has been demonstrated by two methods, Velocity Selective Coherent Population Trapping (VSCPT) [AAK88] and Raman cooling [KaC92].

1.3 Subrecoil cooling and Lévy statistics

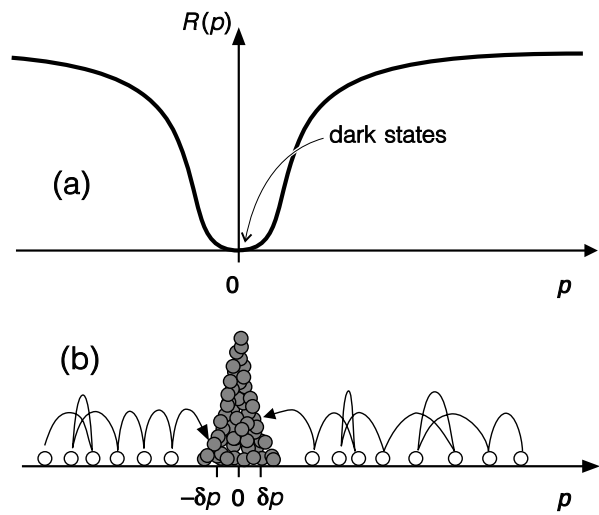


Fig. 1.1. Principle of subrecoil cooling. (a) The fluorescence rate $R(p)$ vanishes at momentum $p = 0$. (b) The atoms perform a random walk in p -space and accumulate in the vicinity of $p = 0$.

Cooling, i.e. an increase of the momentum space density in a narrow range around $\mathbf{p} = \mathbf{0}$, no longer results here from a friction force pushing the atoms towards $\mathbf{p} = \mathbf{0}$, but from a combination of two effects: momentum diffusion and vanishing of the jump rate of the random walk when $\mathbf{p} \rightarrow \mathbf{0}$. Another important difference between this and other cooling schemes is the absence of a steady-state value of the momentum distribution: the longer the interaction time θ , the narrower the range δp around $\mathbf{p} = \mathbf{0}$ in which the atoms can remain trapped during θ . Because of the absence of a steady-state and because of the existence of atomic characteristic times (trapping times) that can be longer than the observation time, here we will call such cooling *non-ergodic cooling*. It has also been called *subrecoil cooling* because nothing now prevents the atomic momentum spread δp reaching values smaller than the photon momentum $\hbar k$.

1.3 Subrecoil cooling and Lévy statistics

We present in this book a new general description of non-ergodic or subrecoil cooling in terms of a competition between trapping processes (i.e. the atom falls in the trap) and recycling processes (i.e. the atom leaves the trap and eventually returns to it). The fundamental feature which has stimulated the new approach presented in this book is that the distributions of trapping times and escape times can be very broad, so broad that the usual *Central Limit Theorem* (CLT) can no longer be applied to study the distributions of the total trapping time and of the

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Excerpt

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total recycling time after N entries in the trap separated by N exits. We show in this book that the so-called *Lévy statistics*, which generalizes the CLT to broad distributions with power-law tails, is the appropriate tool for this problem and that it can provide quantitative results for all the important characteristics of cooled atoms.

Lévy statistics is an outcome of some fundamental mathematical work performed in the 1930's [Lev37, GnK54]. The goal was then to find stable distribution laws for the sum of N independent random variables, i.e. the distribution laws that keep the same mathematical form when $N \rightarrow \infty$. Gaussian distributions and Lévy distributions are the solutions of this problem. While the immense applicability of Gaussian distributions was recognized long ago, Lévy laws have been unduly ignored in the natural sciences and have been considered a sheer mathematical curiosity. However, the situation has completely changed over the last 15 years. Lévy statistics is now recognized as the best tool for studying many anomalous diffusion problems for which standard statistics are inadequate. Application fields include not only physics (anomalous diffusion, chaotic dynamics, mechanics of sandpiles, . . .) but also finance, biology, etc. (see [BoG90, SZF95, Bak96, BCK97, Man97, Zas99, MaS99, PaB99, BoP00, CoR00, GoL01]). Lévy statistics can handle situations in which the standard deviation (or even the average value) of the studied random variable does not exist. It provides technical tools for performing calculations. Importantly, Lévy statistics implies properties that depart very strongly, not only quantitatively but even qualitatively, from usual statistical behaviour. For instance, when the average value of a random variable x is infinite, the sum $\sum_{i=1}^N x_i$ is no longer proportional to the number N of terms (usual law of large numbers), but a different *scaling* behaviour is obtained. This of course has dramatic phenomenological consequences, as we will see for the specific case of subrecoil laser cooling.

From the point of view of laser cooling, the study of subrecoil cooling by Lévy statistics turns out to be extremely fruitful. First, it allows one to extract the key ingredients of the cooling process from the relatively complicated microscopic description of the problem provided by atomic physics. Moreover, the statistical approach leads to unique analytical predictions for the asymptotic properties of the cooled atoms, independent of the details of the particular cooling scheme considered, as expected when one goes from a microscopic description to a statistical description.

From the point of view of statistics, this work can also be considered as a case study for the application of Lévy statistics in a privileged situation where the statistical model can be derived from first principles, developed analytically and, finally, precisely compared to microscopic theoretical approaches and to experimental results.

1.4 Content of the book

This book is intended for two different communities working in two different fields: atomic physics and quantum optics on the one hand and statistical processes on the other. We have thus considered it useful to include a summary of important results already known to each community, but not necessarily by both. These basic results are presented in Chapter 2 (laser cooling, see also Appendix A) and Chapter 4 (Lévy statistics), while Chapter 3 introduces the models that connect both fields. We then proceed in Chapters 5 and 6 with the derivation of the central results of this work. These results are then interpreted, discussed and extended in Chapters 7, 8 and 9. Appendices present several technical developments, either on Lévy statistics, or on subrecoil cooling processes.

More precisely, in *Chapter 2*, we recall some atomic physics results on laser cooling and subrecoil cooling. We point out the difficulties of an exact quantum treatment of subrecoil cooling using the generalized optical Bloch equations, and we present a more efficient quantum approach based on stochastic descriptions of the evolution of the wave function of the system, in terms of quantum jumps occurring at random times. This approach provides Monte Carlo simulations of the quantum evolution of the atomic momentum which allow one to describe, in a rigorous way, the cooling process by inhomogeneous random walks in momentum space with a momentum-dependent jump rate $R(p)$ vanishing for $p = 0$.

Such an approach suggests a simplified model where we make a partition between two classes of atoms: (i) the cold atoms, which are in a trapping volume in momentum space where the momentum $\mathbf{p}$ is close to zero, and which stay for a long time τ (trapping time) in this trapping volume (trapped atoms); (ii) atoms outside of the trapping volume, which make a random walk of duration $\hat{\tau}$ in momentum space, under the effect of radiation, until they come back again in the trapping volume ($\hat{\tau}$ is a first return time). We calculate in *Chapter 3* the probability distributions $P(\tau)$ and $\hat{P}(\hat{\tau})$ of the trapping times and first return times, and we show that in several important cases these distributions are broad distributions with power-law tails, for which Lévy statistics provides the relevant statistical treatment.

Chapter 4 summarizes the main results of Lévy statistics needed for the derivation and the interpretation of the results presented in this work. We will not give here all the detailed proofs, but rather emphasize the physical meaning of the results and the important differences between Lévy statistics and usual Gaussian statistics. More details may be found in [GnK54, Lev37, BoG90]. We also introduce a ‘sprinkling distribution’ which will be the basic tool for the calculations of the following chapters.

The concepts introduced in Chapter 4 are used in the following chapters for the derivation of quantitative predictions concerning laser cooling. We first study, in

Chapter 5, the proportion $f_{\text{trap}}(\theta)$ of cooled atoms after a cooling time θ , for the various cases considered in *Chapter 3*. We find not only the asymptotic behaviour of $f_{\text{trap}}(\theta)$, but the rate at which this asymptotic behaviour is reached when $\theta \rightarrow \infty$. This allows us to give a first characterization of the efficiency of the cooling process. An important result of this chapter is also that $f_{\text{trap}}(\theta)$, defined here as an ensemble average, can be different from the corresponding time average. This clearly shows the *non-ergodic* character of the laser cooling process considered here.

A further step is achieved in *Chapter 6* by calculating the momentum distribution $\mathcal{P}(\mathbf{p})$ of the cold (trapped) atoms and the momentum distribution $\pi(p)$ along a given axis. We show that there is always in $\pi(p)$ a narrow peak whose width δp tends to zero when $\theta \rightarrow \infty$, and the fraction of atoms contained in this peak is calculated in several important cases. The tails of $\pi(p)$ at large p are also studied. Their decrease is described by a power law, which shows that $\pi(p)$ is not a Gaussian distribution so that it is not possible, strictly speaking, to define a thermodynamic temperature. Finally, the increase of the density of atoms in momentum space and in phase space is evaluated for the various situations considered in this work.

The physical content of the results obtained in the preceding chapters is discussed in detail in *Chapter 7*. We re-interpret them in terms of rate equations describing a competition between a rate of entry in the trapping volume and a rate of departure. The rate of entry $S_R(t)$ is in fact nothing but the ‘sprinkling distribution’ introduced in *Chapter 4*. This enables one to interpret the behaviour of the height and of the peak of the momentum distribution. We also discuss in *Chapter 7* a few other problems: the effect of a non-vanishing jump rate when $p \rightarrow 0$ and the connection between non-stationarity, non-ergodicity and broad distributions.

In *Chapter 8* we compare the analytical predictions of the statistical approach presented in this book with experimental results, as well as with predictions of microscopic theoretical approaches based on microscopic quantum treatments (stochastic wave function simulations or generalized optical Bloch equations). The excellent agreement between the various results gives us confidence in the approach developed in this work and in the approximations upon which it is based.

In *Chapter 9* we present an example of application of the approach developed in this book to a specific problem: the optimization of the height of the peak of cooled atoms. This brings into play both the insights and the technical results obtained in previous chapters and deepens our understanding of some properties of non-ergodic cooling.

We finally summarize in *Chapter 10* the main results derived in this book. We also mention a few possible extensions and a few open problems.

2

Subrecoil laser cooling and anomalous random walks

In this chapter, we first recall (in Section 2.1) a few properties of the most usual laser cooling schemes, which involve a friction force. In such standard situations, the motion of the atom in momentum space is a Brownian motion which reaches a steady-state, and the recoil momentum of an atom absorbing or emitting a single photon appears as a natural limit for laser cooling. We then describe in Section 2.2 some completely different laser cooling schemes, based on inhomogeneous random walks in momentum space. These schemes, which are investigated in the present study, allow the ‘recoil limit’ to be overcome. They are associated with non-ergodic statistical processes which never reach a steady-state. Section 2.3 is devoted to a brief survey of various quantum descriptions of subrecoil laser cooling, which become necessary when the ‘recoil limit’ is reached or overcome. The most fruitful one, in the context of this work, is the ‘quantum jump description’ which will allow us in Section 2.4 to replace the microscopic quantum description of subrecoil cooling by a statistical study of a related classical random walk in momentum space. It is this simpler approach that will be used in the subsequent chapters to derive some quantitative analytical predictions, in cases where the quantum microscopic approach is unable to yield precise results, in particular in the limit of very long interaction times, and/or for a momentum space of dimension D higher than 1. This approach also has the advantage of showing, from the beginning, that anomalous random walks and Lévy statistics are deeply involved in subrecoil laser cooling.

2.1 Standard laser cooling: friction forces and the recoil limit

2.1.1 Friction forces and cooling

Laser cooling consists of using resonant exchanges of linear momentum between atoms and photons for reducing the momentum spread δp of an ensemble of atoms. Achieving the smallest possible value of δp has led to spectacular developments

in various research fields such as metrology, high resolution spectroscopy, atom optics, atomic interferometry, Bose–Einstein condensation of atomic gases, . . .

In the most usual laser cooling schemes, an ensemble of atoms interacts with laser beams with suitable polarizations, intensities and frequencies, so that the atomic momenta are damped. In the so-called ‘Doppler cooling’ scheme for instance [HaS75, WiD75], this damping is due to an imbalance between opposite radiation pressure forces, induced by the Doppler effect and resulting in a net force opposed to the atomic motion. In the more efficient ‘Sisyphus cooling’ scheme [DaC89, UWR89, CoP90], atoms run up potential hills (where they decelerate) more frequently than they run down, with the net result of a decrease of atomic kinetic energy.

In all these situations, the cooling effect can be expressed by a friction force which, around $\mathbf{p} = \mathbf{0}$, is proportional to the atomic momentum $\mathbf{p}$ with a negative coefficient [Coh90]. When the friction coefficient is large, atoms seem to move in a very viscous medium, and this kind of situation is called ‘optical molasses’ [CHB85, LPR89].

Momentum damping by a friction force is a dissipative process, necessarily associated with some fluctuations. In the laser cooling schemes considered above, the fluctuations are due to spontaneous emission of fluorescence photons which can be emitted at random times and in random directions, resulting in a random fluctuating component of the momentum exchanged between the atom and the radiation field. Laser cooling mechanisms associated with a friction force therefore give rise to a random walk of the atomic momentum $\mathbf{p}$. As in usual Brownian motion, such a random walk can be characterized by a drift of the atomic momentum towards $\mathbf{p} = \mathbf{0}$ due to the friction force (damping of the mean momentum), and by a momentum diffusion, due to the randomness introduced here by spontaneous emission. Competition between friction and diffusion eventually leads to a steady-state, where the momentum distribution can be characterized by a stationary probability distribution of half-width δp . However, even in the steady-state, fluorescence cycles never cease, and the random walk in momentum space never stops.

Even in the steady-state of the cooling process, laser cooled gases are generally not in thermal equilibrium and, as such, cannot be characterized by a well defined thermodynamic temperature. It is, however, convenient to express the half-width δp of the distribution using an ‘effective temperature’ T defined by

$$\frac{1}{2} k_B T = \frac{\delta p^2}{2M}, \tag{2.1}$$

where k_B is the Boltzmann constant, M is the atomic mass and δp is the half-width at $e^{-1/2}$ of the maximum of the one-dimensional momentum distribution. When the momentum distribution is Maxwell–Boltzmann, the definition of eq. (2.1)

coincides with the usual definition of statistical mechanics. Throughout this book, we will use notations T and δp as defined by eq. (2.1).

2.1.2 The recoil limit

What is the minimum temperature that can be reached by standard laser cooling mechanisms? Since fluorescence cycles never cease and involve spontaneously emitted photons which communicate to the atom a random recoil, it seems difficult to control the momentum spread δp to better than the ‘recoil limit’ $\hbar k$, the momentum of a single photon. In terms of temperature, the recoil limit reads:

$$T \geq T_R, \tag{2.2}$$

where the ‘recoil temperature’ T_R is defined by

$$T_R = \frac{\hbar^2 k^2}{M k_B}. \tag{2.3}$$

The quantity $k_B T_R/2$ is the kinetic energy given to an atom at rest by the absorption or the emission of a single photon. For most atoms, the recoil temperature is of the order of 1 microkelvin (μK). Optimized Sisyphus cooling leads to temperatures close to the recoil limit [CaM95].

2.2 Laser cooling based on inhomogeneous random walks in momentum space

2.2.1 Physical mechanism

It is also possible to achieve laser cooling, i.e. to accumulate atoms around the origin $\mathbf{p} = \mathbf{0}$ of the momentum space, without any friction force. Rather than pushing the atoms towards $\mathbf{p} = \mathbf{0}$ (drift of the momentum random walk associated with the friction), one resorts to an inhomogeneous diffusion coefficient, vanishing around $\mathbf{p} = \mathbf{0}$, so that the random walk slows down in this region, where the atoms pile up. Although random walks seem at first sight to be less efficient at reducing momenta than the deterministic trend provided by friction, it is these random walks that will enable one to circumvent the recoil limit.

Such a situation is achieved when the fluorescence rate R , at which photons are spontaneously re-emitted, depends on $\mathbf{p}$ and exactly vanishes for atoms with zero momentum $\mathbf{p} = \mathbf{0}$ (Fig. 1.1a).

The consequence of the vanishing of the fluorescence rate $R(\mathbf{p})$ around $\mathbf{p} = \mathbf{0}$ is that ultracold atoms ($p \simeq 0$) no longer undergo the random recoils which would be due to spontaneous emissions. They are in some sense protected from the ‘bad’ effects of light. On the other hand, atoms with $p \neq 0$ can absorb

light. The random recoil due to the re-emitted photons modifies in a random way their momentum, which can move closer to zero, or farther from zero, after each fluorescence cycle. The $\mathbf{p}$ -dependent fluorescence rate of Fig. 1.1a is thus at the origin of an inhomogeneous random walk in $\mathbf{p}$ -space, with a $\mathbf{p}$ -dependent jump rate $R(\mathbf{p})$, which eventually transfers atoms from the $p \neq 0$ absorbing states into the $p \simeq 0$ non-absorbing ‘dark’ states where they remain trapped and pile up (Fig. 1.1b).

A simple analogy can help understanding such a physical mechanism. Consider sand grains in a Kundt tube where a resonant acoustic standing wave is excited. The sand grains are vibrating and moving along the axis of the standing wave in an erratic way, except at the nodes of the standing wave, where there is no sound vibration to excite them. After a certain time, the sand grains accumulate at the nodes of the standing wave. In both cases, cooling without friction and Kundt's tube, we have an inhomogenous random walk, i.e. a random walk with a jump rate varying with the location of the particle and vanishing at certain places. For cooling, however, the random walk is in momentum space and the jump rate is momentum-dependent, whereas in a Kundt's tube, the sand grains move in real space, and the jump rate is position-dependent.

The fact that the cooling processes considered here do not rely on a friction force does not mean of course that they could not benefit from the presence of such a friction force. Without friction, the cooling relies on the efficiency with which a pure random walk can bring an atom near $\mathbf{p} = \mathbf{0}$. Such an efficiency decreases dramatically with the number D of dimensions. In one dimension, every particle returns often to the origin of momentum space, although it may take a very long time; in two dimensions, this return takes a much longer time and in three dimensions returning to the origin becomes even more unlikely. It is thus very useful to supplement the momentum random walk with a friction force, producing a drift that tends to push the atoms towards the origin of momentum space [MaA91]. This clearly improves the accumulation process. All subrecoil cooling schemes that have been implemented offer the possibility of an efficient friction force.

2.2.2 How to create an inhomogeneous random walk

Up to now, two methods of laser cooling based on the inhomogeneous random walk presented above have been proposed and demonstrated: Velocity Selective Coherent Population Trapping (VSCPT) [AAK88] and Raman cooling [KaC92]. In this book, we do not present a detailed description of these two methods¹, which can be found elsewhere [AAK89, Coh90, Ari91, OIM90, MaA91, KaC92, Rei96,

¹ However, Appendix A gives a detailed derivation of the parameters of inhomogeneous random walks corresponding to VSCPT and Raman cooling.