

Contents

1	Introduction	<i>page</i> 1
2	Basic material	7
2.1	Yang–Mills theory over compact manifolds	7
2.2	The case of a compact 4-manifold	9
2.3	Technical results	10
2.4	Manifolds with tubular ends	13
2.5	Yang–Mills theory and 3-manifolds	14
	2.5.1 Initial discussion	14
	2.5.2 The Chern–Simons functional	16
	2.5.3 The instanton equation	20
	2.5.4 Linear operators	23
2.6	Appendix A: local models	27
2.7	Appendix B: pseudo-holomorphic maps	30
2.8	Appendix C: relations with mechanics	33
3	Linear analysis	40
3.1	Separation of variables	40
	3.1.1 Sobolev spaces on tubes	45
3.2	The index	47
	3.2.1 Remarks on other operators	51
3.3	The addition property	53
	3.3.1 Weighted spaces	58
	3.3.2 Floer’s grading function; relation with the Atiyah, Patodi, Singer theory	64
	3.3.3 Refinement of weighted theory	68
3.4	L^p theory	70

vi	<i>Contents</i>	
4	Gauge theory and tubular ends	76
4.1	Exponential decay	77
4.2	Moduli theory	82
4.3	Moduli theory and weighted spaces	87
4.4	Gluing instantons	91
	4.4.1 Gluing in the reducible case	100
4.5	Appendix A: further analytical results	103
	4.5.1 Convergence in the general case	103
	4.5.2 Gluing in the Morse–Bott case	108
5	The Floer homology groups	113
5.1	Compactness properties	113
5.2	Floer’s instanton homology groups	122
5.3	Independence of metric	123
5.4	Orientations	130
5.5	Deforming the equations	134
	5.5.1 Transversality arguments	139
5.6	$U(2)$ and $SO(3)$ connections	145
6	Floer homology and 4-manifold invariants	151
6.1	The conceptual picture	151
6.2	The straightforward case	158
6.3	Review of invariants for closed 4-manifolds	161
6.4	Invariants for manifolds with boundary and $b^+ > 1$	165
7	Reducible connections and cup products	168
7.1	The maps D_1, D_2	168
7.2	Manifolds with $b^+ = 0, 1$	169
	7.2.1 The case $b^+ = 1$	171
	7.2.2 The case $b^+ = 0$	174
7.3	The cup product	176
	7.3.1 Algebraic-topological interpretation	176
	7.3.2 An alternative description	179
	7.3.3 The reducible connection	183
	7.3.4 Equivariant theory	188
	7.3.5 Limitations of existing theory	196
7.4	Connected sums	201
	7.4.1 Surgery and instanton invariants	201
	7.4.2 The $\text{Hom}_{\mathcal{F}}$ -complex and connected sums	206
8	Further directions	213
8.1	Floer homology for other 3-manifolds	213

	<i>Contents</i>	vii
8.2	The blow-up formula	219
	<i>Bibliography</i>	231
	<i>Index</i>	235