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in Yang–Mills theory

Floer homology groups in Yang–Mills theory

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Imperial College, London

with the assistance of M. Furuta and D. Kotschick



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