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978-0-521-80795-1 - Restricted Orbit Equivalence for Actions of Discrete Amenable Groups

Janet Whalen Kammeyer and Daniel J. Rudolph

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The purpose of this work is to lift the notion of restricted orbit equivalence to the category of free and ergodic actions of discrete amenable groups. The axiomatics of a size  $m$  and the nature of the associated equivalence relation,  $m$ -equivalence, are established. An extensive list of examples of sizes and the corresponding equivalence relations are described. An entropy, called  $m$ -entropy, associated with each size, is defined as the infimum of the classical entropy over the  $m$ -equivalence class. It is proven that a restricted orbit equivalence is either entropy preserving, in that the  $m$ -entropy is simply the classical entropy, or entropy free, in that on a residual subset of the equivalence class, the entropy is zero and hence the  $m$ -entropy of all actions is zero. The notion of  $m$ -finitely determined is introduced, and some of its basic properties are developed, in particular that it is an  $m$ -equivalence invariant. Finally, the equivalence theorem is proven, that any two  $m$ -finitely determined actions of equal  $m$ -entropy are  $m$ -equivalent. This is carried out using category methods, following the Burton–Rothstein approach, within a natural Polish space of  $m$ -joinings. In the Appendix, it is shown that previous size axiomatizations give rise to essentially the same notion of  $m$ -equivalence.

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