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0521806429 - A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations - David de la Croix and Philippe Michel

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