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0521806429 - A Theory of Economic Growth: Dynamics and Policy in Overlapping Generations - David de la Croix and Philippe Michel

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A Theory of Economic Growth

This book provides an in-depth treatment of the overlapping generations model in economics incorporating production. In chapter 1, the authors investigate competitive equilibria and corresponding dynamics: existence and uniqueness of equilibrium, global dynamics of capital (including poverty traps), and various extensions of the model. Chapter 2 analyzes the optimality of allocations in this framework, using both the value function and marginal approaches. Optimality with unbounded growth is also analyzed. Policy issues, including the Second Welfare Theorem, pensions, government spending, and optimal taxation, are discussed in chapter 3. The notion of public debt is introduced in chapter 4, and the sustainability of policies with budget deficits/surpluses is examined. The last chapter presents extensions of the model including altruism, education/human capital, and habit formation. Methodological emphasis is put on using general preferences and technologies, on the global study of dynamic aspects of the model, and on furnishing adequate tools to analyze policies involving inter-generational transfers.

David de la Croix is Professor of Economics at the Université catholique de Louvain and Research Associate of the National Fund for Scientific Research in Belgium. He has served as a visiting professor at the University of California, Los Angeles. Professor de la Croix's research interests in macro-economics include demographics, overlapping generations and human capital, and growth and cycles. He has published articles in leading refereed journals such as the *Journal of Economic Theory*, *European Economic Review*, *Journal of Economic Dynamics and Control*, *Journal of Applied Economics*, *Review of Economic Dynamics*, and *Journal of Population Economics*.

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Further Praise for *A Theory of Economic Growth*

“The overlapping generations model and the infinitely lived or dynastic model are the two workhorses of modern macroeconomics. De La Croix and Michel have written a wonderfully accessible graduate textbook on the overlapping generations model. They carefully take students through essentially every variant of the model, prove a large number of known results, and offer a few new ones as well. This book is an essential addition as a teaching tool and an invaluable reference on every economist’s shelf.”

– V. V. Chari, *University of Minnesota*

“In recent decades overlapping generation models have become a central framework of analysis in the research of economic growth. The authors present a comprehensive and lucid exposition of the dynamic structure of the basic overlapping generation features with production. Highly recommended for researchers and graduate students in the fields of growth theory and dynamic macroeconomics.”

– Oded Galor, *Brown University*

“Some of the most hotly debated government policies are those that involve redistribution across generations, such as social security and public education. De la Croix and Michel provide a manual of economic tools for evaluating these sorts of policies based on simple overlapping generations models. Their presentation of these models strives to combine applicability for policy analysis with a solid foundation in dynamic general equilibrium theory. Their book should be of use to economists from the level of advanced undergraduate students to researchers and teachers.”

– Timothy J. Kehoe, *University of Minnesota and Federal Reserve Bank of Minneapolis*

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A Theory of Economic Growth

Dynamics and Policy in Overlapping Generations

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*To Anna, Clémence, Eloïse, Françoise,
Mathieu, Thérèse, Timothée,
and all future, yet unborn, generations.*

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Introduction

Inter-generational transfers are today at the center of the economic policy debate. The reduction of public debt, the financing of social security (pensions), the taxation of capital and bequests, and the design of the education system all imply substantial inter-generational transfers.

The tool economists provide to analyze these issues is the overlapping generations model. As it models explicitly the different periods of life – schooling, working, or retirement periods – it is the natural framework to study the allocation of resources across the different generations.

When it includes capital accumulation, this model also allows one to formalize the development of an economy, relating its growth path to the savings behavior of young agents.

The aim of this book is to provide the reader with an in-depth introduction to this model, including its major policy aspects.

OVERLAPPING GENERATIONS AND MACRO-ECONOMICS

Modern macro-economics is generally characterized by four elements: (a) The issues of concern are aggregate in nature. (b) The models in use are derived from optimizing behavior, and, as a consequence, their properties depend essentially on preferences and technologies. (c) Interactions over time are explicitly taken into account, giving therefore an important place to dynamic analysis. (d) The general equilibrium framework is preferred to partial equilibrium setups.

The building blocks of modern macro-economics are taken from two different approaches: The first one considers that agents have an infinite horizon. The second one analyzes the case of an economy in which agents have finite lives. This second approach consists in the so-called overlapping generations models. The central mechanics of this class of model are the decisions of young agents about how much to consume and save for retirement, i.e., the life-cycle hypothesis of savings (see Ando and Modigliani (1963)).

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A series of issues are common to both approaches, like the analysis of the factors influencing economic growth. Several issues are, however, specific to the overlapping generations approach. In general they are related to inter-generational redistribution, and hence, e.g., to social security, education policies, and public debt questions. Indeed, even in its simplest version, the overlapping generations model embeds at least two types of agents living at the same time, young and old, which makes possible an analysis of distributional issues.

Moreover, the model with a representative infinite-lived agent can be seen as a special case of the overlapping generations model where households are altruistic and care about their descendants: When altruism is strong enough so that every generation leaves positive bequests, the properties of the two models are the same.

Another domain of research heavily involving the overlapping generations model is the quest for reasonable mechanisms of endogenous fluctuations. Although, as we shall see, the basic model is characterized by monotonic dynamics, various extensions lead to oscillatory dynamics, or even, in extreme cases, permanent endogenous cycles.

When one uses overlapping generations models with two-period-lived agents, the unit of time that one considers is of the order of 20 or 30 years. One could be tempted to think that the results obtained in a simple benchmark model with this periodicity can be generalized to n -period-lived agents, and the conclusions of the reference model applied to questions relevant at the business cycle frequency.¹ In this case, the model with two-period-lived agents is a metaphor. One should be cautious, however, as all the properties of a model with two-period-lived agents cannot always be extended to n -period-lived agents.

Three important properties of overlapping generations models are the non-neutrality of debt, the possibility of asset bubbles, and the possibility for competitive equilibria to be inefficient. Weil (1989) shows that they do not depend on the horizon (finite or infinite) of the agents. However, the effect of interest rate on saving behavior and the type of dynamics (monotonous, oscillatory, etc.) do depend crucially on the number of periods of life considered. It should then be clear that the overlapping generations model in its standard form² has little to say on short-run issues. For this reason, we shall concentrate our attention on long-term problems involving inter-generational transfers.

¹ Assuming that workers do not have access to financial markets, Woodford (1986) develops an infinite horizon model with the same structure as an overlapping generations model with two-period-lived agents.

² The model of perpetual youth due to Blanchard (1985), which is at the intersection of infinite horizon models and overlapping generations models, can be used to describe phenomena at the business-cycle frequency. This model was further extended by Saint-Paul (1992) to allow for endogenous growth, and by Frenkel and Razin (1986) to an open economy setup.

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Beyond the fact that it is a framework with heterogeneous agents, the overlapping generations model presents two attractive features: First, it focuses on the life-cycle behavior of the agents and hence on their savings behavior as a function of age. Second, it is a model in which the competitive equilibrium is not necessarily Pareto optimal.

OVERLAPPING GENERATIONS IN OTHER FIELDS

There has been an extensive literature in public economics that uses the overlapping generations model. The issues at stake concern the design of optimal tax schemes when the government cannot use non-distortionary lump-sum instruments. The study of bequest taxation and capital taxation has a central role. Education funding through public or private institutions is another typical topic.

Every time policy changes affect different generations in different ways, the overlapping generations model is useful. For example, it is used in environmental economics, as pollution control might harm current generations but promote the welfare of the not yet born generations. Population economics is another example, where authors study the effect of rising longevity, lowered fertility, and population dynamics on human and physical capital accumulation.

Development economics and growth theory also rely on overlapping generations. There are several reasons. First, if one believes that human capital is a relevant factor of development and growth, modeling the education process is important. As education usually takes place at the beginning of the life cycle and involves transfers from one generation to the other, a structure with overlapping generations is appealing. Second, one property of the simplest overlapping generations model is to make long-term growth dependent on initial conditions. In other words, the starting point of the economy is crucial to determining its future. For instance, starting with too low capital might lead the economy into a poverty trap. This dependence on history through initial conditions is important for understanding the different development patterns observed all over the world.

In monetary theory, money has been considered as a medium of exchange between generations and as a store of value. In particular, in the absence of physical capital, money is useful per se, as it allows households to transfer resources across periods.

Finally, mathematical economics have used the overlapping generations model to study under which conditions the equilibrium is indeterminate. Following the logic of Arrow and Debreu, when all markets of all periods clear at once, there is a continuum of equilibria in a context where neither the passage of time nor the formation of expectations is central to the analysis.

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Introduction

OUTLINE OF THE BOOK

Many advanced graduate texts that treat one-sector growth models tackle the basic overlapping generations model. But it is difficult to find, in one single monograph, a survey of the main results, analyzed in detail, as well as extensions of some of the key results in the literature. The extensive analysis of the key models makes this text useful for applied theorists, including researchers in macro-economics, public finance, and development-growth theory, who wish to apply the overlapping generations framework.

The book is also intended for students at the graduate level. True, it is not an easy book for a graduate student to tackle, at least not at the start. In any case, this slot of the textbook market is already well served by general-purpose macro textbooks. But our text adds another dimension: It brings together the standard overlapping generations model with its policy implications (pension funding, debt policy) in one place. It also provides results that have never been demonstrated in the framework considered. We hope that it will be popular with advanced graduate students who have chosen to work in the area of macro dynamics.

This book contains five chapters. The first four chapters cover an analysis of the basic overlapping generations model. One goal that we pursue is to provide the reader with a set of propositions deriving the properties of the overlapping generations model when we make few assumptions on preference and technology. This should help students and researchers in their modeling choices. In chapter 1 we propose an in-depth study of competitive equilibria in the basic overlapping generations model. Chapter 2 is devoted to the analysis of optimality in this setup. Policy issues, including pensions and optimal taxation, are discussed in chapter 3. Public debt is introduced in chapter 4, and its sustainability is analyzed. The last chapter includes various extensions, including altruism, education, and habit formation. The technical points are detailed in the appendix.

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