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The Estimation and Tracking of Frequency

B. G. Quinn

University of Manchester Institute of Science and Technology

E. J. Hannan

Late, Australian National University

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To Margie, Tim, Jamie, Nicky and Robbie
and to the memory of Ted.

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Preface

In late 1982, Ted Hannan discussed with me a question he had been asked by some astronomers – how could you estimate the two frequencies in two sinusoids when the frequencies were so close together that you could not tell, by looking at the periodogram, that there *were* two frequencies? He asked me if I would like to work with him on the problem and gave me a reprint of his paper (Hannan 1973) on the estimation of frequency. Together we wrote a paper (Hannan and Quinn 1989) which derived the regression sum of squares estimators of the frequencies, and showed that the estimators were strongly consistent and satisfied a central limit theorem. It was clear that there were no problems asymptotically if the two frequencies were fixed, so Ted's idea was to fix one frequency, and let the other converge to it at a certain rate, in much the same way as the alternative hypothesis is constructed to calculate the asymptotic power of a test. Since then, I have devoted much of my research to sinusoidal models. In particular, I have spent a lot of time constructing algorithms for the estimation of parameters in these models, to implementing the algorithms in practice and, for me perhaps the most challenging, establishing the asymptotic (large sample) properties of the estimators.

We commenced writing this book in 1992, while I was working for the Australian Department of Defence as a scientist specialising in underwater signal processing. Our aim was to provide a comprehensive examination of various techniques for estimating fixed and varying frequency without dwelling too much on the minutiae of the theory. We hoped that the book would be accessible to both statisticians and engineers. I wrote what I thought would be the most straightforward chapter, on my work with Jose Fernandes, who had been a PhD student in the Electrical Engineering Department at the University of Newcastle, NSW, Australia. In late 1993, Ted wrote to me, concerned that the book was moving too slowly, and worried that it might

never be completed. He died on January 7th, 1994, having completed several drafts of Chapters 2 and 3. Although I have corrected and added to these chapters, I have adjusted neither the style in which Ted wrote, nor his turn of phrase. It has taken me more than 6 years to complete and revise the book, for a number of reasons, among which have been several changes of location, from the Defence Science and Technology Organisation (DSTO) in Salisbury, Adelaide, Australia to Goldsmiths' College, University of London, and to the University of Manchester Institute of Science and Technology (UMIST).

This book is reasonably self-contained. We begin by motivating the problems by appealing to a small number of physical examples. For this reason, and since some of the important examples are not mentioned elsewhere, it is recommended that all readers consider the Introduction first. While there is also a fair amount of theory there, most theoretical issues are left until Chapter 2, which contains the statistical and probability theory needed for an understanding of the remainder of the book. Chapter 3 is concerned with the inference for fixed frequencies, including the problem of estimating two close frequencies and the estimation of the number of sinusoids. In Chapter 4, we present the Quinn and Fernandes (1991) technique. Unlike most techniques in the book, which involve Fourier transforms, this technique uses sequences of linear filters. In Chapter 5, we consider several popular techniques based on autocovariances, discussing their asymptotic properties and conditions under which they exhibit reasonable behaviour. In Chapter 6 we look at techniques which use only the Fourier transform of a series at the Fourier frequencies (the so-called Fourier coefficients) to estimate the system parameters. Finally, in Chapter 7, we treat the problem of changing frequency – a class of hidden Markov Model (HMM) procedures for tracking frequency as it changes slowly in very low signal-to-noise environments. Readers looking for a quick introduction to the estimation and related problems, and further applications are referred to the excellent expository article by Brillinger (1987).

I would like to thank the many people who have directly or indirectly encouraged our writing of this book. In particular, I would like to thank Dawei Huang (Queensland University of Technology), Bob Bitmead, Peter Hall and Joe Gani (Australian National University), Ross Barrett, Vaughan Clarkson, David Liebing and Darrell McMahon (DSTO, Salisbury), Doug Gray (DSTO and University of Adelaide), Brian Ferguson (DSTO, Sydney), Peter Kootsookos (Cooperative Research Centre for Robust and Adaptive Systems and e-Muse Corporation, Dublin, Ireland), Stephen Searle (Cooperative Research Centre for Robust and Adaptive Systems), Peter Thomson

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Finally, I would like to acknowledge my debt to Ted Hannan, who, after supervising my PhD, introduced me to the topic of the book and encouraged me in every aspect of my early career. I have missed him greatly, both personally and professionally, and I am sure that he would not mind much my dedication of this book to his memory. An account of Ted's life may be found at http://www.asap.unimelb.edu.au/bsparcs/aasmemoirs/hannan.htm, written by his good friend Joe Gani.

Manchester, U.K. Barry Quinn
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