Introduction to Hydrodynamic Stability

Instability of flows and their transition to turbulence are widespread phenomena in engineering and the natural environment, and are important in applied mathematics, astrophysics, biology, geophysics, meteorology, oceanography and physics as well as engineering. This is a textbook to introduce these phenomena at a level suitable for a graduate course, by modelling them mathematically, and describing numerical simulations and laboratory experiments. The visualization of instabilities is emphasized, with many figures, and in references to more still and moving pictures. The relation of chaos to transition is discussed at length. Many worked examples and exercises for students illustrate the ideas of the text. Readers are assumed to be fluent in linear algebra, advanced calculus, elementary theory of ordinary differential equations, complex variables and the elements of fluid mechanics. The book is aimed at graduate students but will also be very useful for specialists in other fields.

Philip Drazin (1934–2002) was Professor of Applied Mathematics at the University of Bristol (1981–1999) and Professor of Mathematical Sciences at the University of Bath (1999–2002). He was the author of many books including *Hydrodynamic Stability* which he co-authored with W. H. Reid (Cambridge University Press, 1981).

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From the Editors

We are deeply saddened to note the death of Philip Drazin during the production of this, his last textbook for the Cambridge Texts in Applied Mathematics. Philip was a wonderful teacher, a superb applied mathematician and an inspiring colleague. During his life he produced seminal work on hydrodynamic stability, particularly applied to problems in meteorology. He was always concerned to understand the mathematics behind the physical problem he was studying, but was always aware of its limitations, and the need to compare mathematical predictions against physical reality.

This book is a fitting tribute to Philip's whole approach to his work. It reflects both his deep understanding of the way mathematics can be applied to natural phenomena and his unique way of illuminating any topic. All who knew him will see his spirit and humour shining through these pages and will benefit from the experience and wisdom he gained by studying many significant practical and theoretical problems. Philip wrote two earlier textbooks within this series, each going on to become classics in their fields. We are sure that this will do the same.

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P. G. DRAZIN

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Preface

This text arose from notes on lectures delivered to M.Sc. students at the University of Bristol in the 1980s. The notes were revised and printed for a course of lectures delivered to postgraduates at the University of Tokyo in 1995. The latter course led to collaboration with Professor Tsutomu Kambe in writing in Japanese the book *Ryutai Rikigaku – Anteisei To Ranyu (Fluid Dynamics – Stability and Turbulence)*, published by the University of Tokyo Press in 1998. The present book is an enlargement in English of the first part of the Japanese book. An advanced draft was prepared for a lecture course given to undergraduates and postgraduates at the University of Oxford in 2001. I am grateful to the many students, at Bristol, Tokyo and Oxford, for their stimulating me to clarify both my ideas and their expression, and their encouragement to learn more. I am especially grateful to Professor Kambe for what I learnt from him and put into the text.

The result is a textbook, not a research monograph. To be sure, many points of current research have been incorporated in the text, but there has been no attempt to lead the reader up to the frontier of current research. So the mathematical theory has been described as simply and briefly as was felt possible, and plenty of worked examples and accessible exercises for students have been included. I have cited many publications, perhaps because the habit of doing so is deeply ingrained, certainly not because I ever imagined that many students care about references, let alone follow them up. The overt intention of including the references is to encourage students' instructors to follow up various details and, most importantly, use still and moving pictures to supplement this book in their teaching.

Indeed, wherever practical, pictures of relevant fluid mechanical experiments are used in the text. This is done primarily by inclusion of illustrations in this book. However, practical limitations of space have led to supplementation of the illustrations in this book by citing other sources, notably the beautiful books *An Album of Fluid Motion*, edited by Van Dyke (1982), and *Visualized Flow*, edited by Nakayama (1988). But hydrodynamic instability is essentially dynamic, so motion pictures and videos can convey many things which still pictures cannot. Accordingly, reference is often made to the wonderful classic series of film loops and motion pictures of the National Education Center; they are old and no longer for sale, but they have been re-issued as videos by the Encyclopedia Britannica Corporation. Further, *Multi-media Fluid Mechanics*, a compact disk by Homsy *et al*. (CD2000), has been published recently. Its section *Video Library* has many short videos relevant to this book, and they are cited in the text. I hope that further videos will be added to the CD in future editions, and am confident that advances in computer technology will soon lead to more such pictorial aids to this book.

It is assumed that readers of this book are familiar with the elements of the theory and practice of fluid mechanics – the material that is included in typical first courses on the motion of inviscid and viscous fluids. So the theory of Euler's equations of motion, irrotational flow, vorticity, the Navier–Stokes equations, boundary-layer theory, separation, and so forth will be used with little explanation wherever they are needed in the text. Again, the elementary theory of linear algebra, complex variables, and ordinary and partial differential equations will be assumed, and used freely as needed. Sections, paragraphs and exercises that demand more advanced knowledge or touch deep matters are preceded by asterisks.

I thank Professor William H. Reid for his generosity in allowing me to reproduce with little alteration Sections 1, 4 and 5 of our book *Hydrodynamic Stability* as respectively Section 1.1, Chapter 3 and Chapter 4 of this book, as well as several exercises. I also thank him for the enormous amount about writing books as well as about hydrodynamic stability which I have learnt from him over many decades.

I thank Professor Herbert E. Huppert, Dr Richard R. Kerswell and Professor Stephen D. Mobbs for suggesting ideas which have led to exercises in this book.

I thank Dr Alan McAlpine for material for Figure 8.10.

I thank Dr Álvaro Meseguer and Professor L. Nicholas Trefethen for their constructive comments on parts of a draft of the book, and for copies of Figures 8.14 and 8.15.

I thank Professor William Saric for an illuminating discussion of the flow past a flat plate.

I thank the reproduction-rights holders for their generous permission to reproduce many of the figures in this book, and to the authors who

have kindly expressed their approval of the reproduction of their original figures.

Philip Drazin
University of Bath
July 2001