CONSISTENT QUANTUM THEORY

Quantum mechanics is one of the most fundamental yet difficult subjects in modern physics. In this book, nonrelativistic quantum theory is presented in a clear and systematic fashion that integrates Born's probabilistic interpretation with Schrödinger dynamics.

Basic quantum principles are illustrated with simple examples requiring no mathematics beyond linear algebra and elementary probability theory, clarifying the main sources of confusion experienced by students when they begin a serious study of the subject. The quantum measurement process is analyzed in a consistent way using fundamental quantum principles that do not refer to measurement. These same principles are used to resolve several of the paradoxes that have long perplexed quantum physicists, including the double slit and Schrödinger's cat. The consistent histories formalism used in this book was first introduced by the author, and extended by M. Gell-Mann, J.B. Hartle, and R. Omnès.

Essential for researchers, yet accessible to advanced undergraduate students in physics, chemistry, mathematics, and computer science, this book may be used as a supplement to standard textbooks. It will also be of interest to physicists and philosophers working on the foundations of quantum mechanics.

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To the memory of my parents
examples of integrity and Christian service

Contents

Preface

Quantum theory is one of the most difficult subjects in the physics curriculum. In part this is because of unfamiliar mathematics: partial differential equations, Fourier transforms, complex vector spaces with inner products. But there is also the problem of relating mathematical objects, such as wave functions, to the physical reality they are supposed to represent. In some sense this second problem is more serious than the first, for even the founding fathers of quantum theory had a great deal of difficulty understanding the subject in physical terms. The usual approach found in textbooks is to relate mathematics and physics through the concept of a *measurement* and an associated wave function collapse. However, this does not seem very satisfactory as the foundation for a fundamental physical theory. Most professional physicists are somewhat uncomfortable with using the concept of measurement in this way, while those who have looked into the matter in greater detail, as part of their research into the foundations of quantum mechanics, are well aware that employing measurement as one of the building blocks of the subject raises at least as many, and perhaps more, conceptual difficulties than it solves.

It is in fact not necessary to interpret quantum mechanics in terms of measurements. The primary mathematical constructs of the theory, that is to say wave functions (or, to be more precise, subspaces of the Hilbert space), can be given a direct physical interpretation whether or not any process of measurement is involved. Doing this in a consistent way yields not only all the insights provided in the traditional approach through the concept of measurement, but much more besides, for it makes it possible to think in a sensible way about quantum systems which are not being measured, such as unstable particles decaying in the center of the earth, or in intergalactic space. Achieving a consistent interpretation is not easy, because one is constantly tempted to import the concepts of classical physics, which fit very well with the mathematics of classical mechanics, into the quantum domain where they sometimes work, but are often in conflict with the very different mathematical structure of Hilbert space that underlies quantum theory. The result of using classical concepts where they do not belong is to generate contradictions and paradoxes of the sort which, especially in more popular expositions of the subject, make quantum physics seem magical. Magic may be good for entertainment, but the resulting confusion is not very helpful to students trying to understand the subject for the first time, or to more mature scientists who want to apply quantum principles to a new domain where there is not yet a well-established set of principles for carrying out and interpreting calculations, or to philosophers interested in the implications of quantum theory for broader questions about human knowledge and the nature of the world.

The basic problem which must be solved in constructing a rational approach to quantum theory that is not based upon measurement as a fundamental principle is to introduce probabilities and stochastic processes as part of the foundations of the subject, and not just an *ad hoc* and somewhat embarrassing addition to Schrödinger's equation. Tools for doing this in a consistent way compatible with the mathematics of Hilbert space first appeared in the scientific research literature about fifteen years ago. Since then they have undergone further developments and refinements although, as with almost all significant scientific advances, there have been some serious mistakes on the part of those involved in the new developments, as well as some serious misunderstandings on the part of their critics. However, the resulting formulation of quantum principles, generally known as *consistent histories* (or as *decoherent histories*), appears to be fundamentally sound. It is conceptually and mathematically "clean": there are a small set of basic principles, not a host of *ad hoc* rules needed to deal with particular cases. And it provides a rational resolution to a number of paradoxes and dilemmas which have troubled some of the foremost quantum physicists of the twentieth century.

The purpose of this book is to present the basic principles of quantum theory with the probabilistic structure properly integrated with Schrödinger dynamics in a coherent way which will be accessible to serious students of the subject (and their teachers). The emphasis is on physical interpretation, and for this reason I have tried to keep the mathematics as simple as possible, emphasizing finite-dimensional vector spaces and making considerable use of what I call "toy models." They are a sort of quantum counterpart to the massless and frictionless pulleys of introductory classical mechanics; they make it possible to focus on essential issues of physics without being distracted by too many details. This approach may seem simplistic, but when properly used it can yield, at least for a certain class of problems, a lot more physical insight for a given expenditure of time than either numerical calculations or perturbation theory, and it is particularly useful for resolving a variety of confusing conceptual issues.

An overview of the contents of the book will be found in the first chapter. In brief, there are two parts: the essentials of quantum theory, in Chs. 2–16, and a variety of applications, including measurements and paradoxes, in Chs. 17–27. References to the literature have (by and large) been omitted from the main text, and will be found, along with a few suggestions for further reading, in the bibliography. In order to make the book self-contained I have included, without giving proofs, those essential concepts of linear algebra and probability theory which are needed in order to obtain a basic understanding of quantum mechanics. The level of mathematical difficulty is comparable to, or at least not greater than, what one finds in advanced undergraduate or beginning graduate courses in quantum theory.

That the book is self-contained does not mean that reading it in isolation from other material constitutes a good way for someone with no prior knowledge to learn the subject. To begin with, there is no reference to the basic phenomenology of blackbody radiation, the photoelectric effect, atomic spectra, etc., which provided the original motivation for quantum theory and still form a very important part of the physical framework of the subject. Also, there is no discussion of a number of standard topics, such as the hydrogen atom, angular momentum, harmonic oscillator wave functions, and perturbation theory, which are part of the usual introductory course. For both of these I can with a clear conscience refer the reader to the many introductory textbooks which provide quite adequate treatments of these topics. Instead, I have concentrated on material which is not yet found in textbooks (hopefully that situation will change), but is very important if one wants to have a clear understanding of basic quantum principles.

It is a pleasure to acknowledge help from a large number of sources. First, I am indebted to my fellow consistent historians, in particular Murray Gell-Mann, James Hartle, and Roland Omnès, from whom I have learned a great deal over the years. My own understanding of the subject, and therefore this book, owes much to their insights. Next, I am indebted to a number of critics, including Angelo Bassi, Bernard d'Espagnat, Fay Dowker, GianCarlo Ghirardi, Basil Hiley, Adrian Kent, and the late Euan Squires, whose challenges, probing questions, and serious efforts to evaluate the claims of the consistent historians have forced me to rethink my own ideas and also the manner in which they have been expressed. Over a number of years I have taught some of the material in the following chapters in both advanced undergraduate and introductory graduate courses, and the questions and reactions by the students and others present at my lectures have done much to clarify my thinking and (I hope) improve the quality of the presentation.

I am grateful to a number of colleagues who read and commented on parts of the manuscript. David Mermin, Roland Omnès, and Abner Shimony looked at particular chapters, while Todd Brun, Oliver Cohen, and David Collins read drafts of the entire manuscript. As well as uncovering many mistakes, they made a large number of suggestions for improving the text, some though not all of which I adopted. For this reason (and in any case) whatever errors of commission or omission are present in the final version are entirely my responsibility.

I am grateful for the financial support of my research provided by the National Science Foundation through its Physics Division, and for a sabbatical year from my duties at Carnegie-Mellon University that allowed me to complete a large part of the manuscript. Finally, I want to acknowledge the encouragement and help I received from Simon Capelin and the staff of Cambridge University Press.

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