
Contents

1	Introduction	<i>page 1</i>
	Part one: TORSORS	7
2	Torsors: general theory	11
2.1	Torsors over a field	11
2.2	Torsors and Čech cohomology	13
2.3	Torsors under groups of multiplicative type	24
2.4	Obstructions to existence of rational points over arbitrary fields	34
3	Examples of torsors	42
3.1	Torsors in geometric invariant theory	42
	Appendix. Classification of Del Pezzo surfaces of degree 5	44
3.2	Homogeneous spaces and central extensions	49
3.3	Torsors under abelian varieties	54
	Appendix. Equations for 2- and 4-coverings of elliptic curves	58
4	Abelian torsors	62
4.1	From abelian torsors to Azumaya algebras	62
4.2	A commutative diagram	68
4.3	Local description of abelian torsors	72
4.4	Torsors associated with a dominant morphism to \mathbf{P}_k^1	79
	Part two: DESCENT AND MANIN OBSTRUCTION	95
5	Obstructions over number fields	98
5.1	The Hasse principle, weak and strong approximation	98
5.2	The Manin obstruction	101
5.3	Descent obstructions	104
6	Abelian descent and Manin obstruction	112
6.1	Descent theory	113

viii	<i>Contents</i>	
6.2	Manin obstruction and global duality pairings	121
6.3	Compactifications of torsors under tori	130
7	Abelian descent on conic bundle surfaces	134
7.1	Brauer group of conic bundles	135
7.2	Châtelet surfaces	139
7.3	Some intersections of two quadrics in \mathbb{P}_k^5	146
7.4	Conic bundles with six singular fibres	148
8	Non-abelian descent on bielliptic surfaces	157
8.1	Beyond the Manin obstruction	158
	Appendix. An example of 4-torsion in $\text{III}(E)$	163
8.2	Interpretation in terms of non-abelian torsors	164
9	Homogeneous spaces and non-abelian cohomology	168
9.1	Liens and non-abelian H^2	168
9.2	The Springer class of a homogeneous space	172
9.3	Abelianization of non-abelian H^2	173
9.4	Hasse principle for non-abelian H^2	174
9.5	Descent on homogeneous spaces	175
	<i>References</i>	179
	<i>Index</i>	187