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978-0-521-80237-6 - Torsors and Rational Points
Alexei Skorobogatov
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CAMBRIDGE TRACTS IN MATHEMATICS

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144 Torsors and Rational Points

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PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
 The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
 The Edinburgh Building, Cambridge CB2 2RU, UK
 40 West 20th Street, New York, NY 10011-4211, USA
 10 Stamford Road, Oakleigh, Melbourne 3166, Australia
 Ruiz de Alarcón 13, 28014 Madrid, Spain
 Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

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First published 2001

Printed in the United Kingdom at the University Press, Cambridge

Typeface Computer Modern 10/13pt *System* LaTeX2e [UPH]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

Skorobogatov, Alexei, 1961-
 Torsors and rational points / Alexei Skorobogatov
 p. cm. – (Cambridge tracts in mathematics; 144)
 Includes bibliographical references and index.

ISBN 0 521 80237 7

1. Torsion theory (Algebra) I. Title.

QA251.3.S62 2001

512.4'–dc21 2001025430

ISBN 0 521 80237 7 hardback

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Вещи все превращены.
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Как феномены и сны.

Возникает мир чудесный
В человеческом мозгу.
Он течет водою пресной
Разгонять твою тоску.

...

На кустах сидят сомненья
В виде галок и ворон,
В деревьях - столпотворенье
Чисел, символов, имен.

Перед бабочкой пучина
Неразгаданных страстей...
Геометрия - причина
Прорастания стеблей.

Н. М. Олейников. "Пучина страстей" (философская поэма), Финал.

Contents

1	Introduction	<i>page 1</i>
	Part one: TORSORS	7
2	Torsors: general theory	11
2.1	Torsors over a field	11
2.2	Torsors and Čech cohomology	13
2.3	Torsors under groups of multiplicative type	24
2.4	Obstructions to existence of rational points over arbitrary fields	34
3	Examples of torsors	42
3.1	Torsors in geometric invariant theory	42
	Appendix. Classification of Del Pezzo surfaces of degree 5	44
3.2	Homogeneous spaces and central extensions	49
3.3	Torsors under abelian varieties	54
	Appendix. Equations for 2- and 4-coverings of elliptic curves	58
4	Abelian torsors	62
4.1	From abelian torsors to Azumaya algebras	62
4.2	A commutative diagram	68
4.3	Local description of abelian torsors	72
4.4	Torsors associated with a dominant morphism to \mathbf{P}_k^1	79
	Part two: DESCENT AND MANIN OBSTRUCTION	95
5	Obstructions over number fields	98
5.1	The Hasse principle, weak and strong approximation	98
5.2	The Manin obstruction	101
5.3	Descent obstructions	104
6	Abelian descent and Manin obstruction	112
6.1	Descent theory	113

viii	<i>Contents</i>	
6.2	Manin obstruction and global duality pairings	121
6.3	Compactifications of torsors under tori	130
7	Abelian descent on conic bundle surfaces	134
7.1	Brauer group of conic bundles	135
7.2	Châtelet surfaces	139
7.3	Some intersections of two quadrics in \mathbb{P}_k^5	146
7.4	Conic bundles with six singular fibres	148
8	Non-abelian descent on bielliptic surfaces	157
8.1	Beyond the Manin obstruction	158
	Appendix. An example of 4-torsion in $\text{III}(E)$	163
8.2	Interpretation in terms of non-abelian torsors	164
9	Homogeneous spaces and non-abelian cohomology	168
9.1	Liens and non-abelian H^2	168
9.2	The Springer class of a homogeneous space	172
9.3	Abelianization of non-abelian H^2	173
9.4	Hasse principle for non-abelian H^2	174
9.5	Descent on homogeneous spaces	175
	<i>References</i>	179
	<i>Index</i>	187